EKSAMEN NOV. 2017 ①

1. a) $f(x) = 2x^3 + 2x^2 - 34x + 30$

$$2x^3 + 2x^2 - 34x + 30 : x + 5 = \underline{\underline{2x^2 - 8x + 6}}$$
$$-(2x^3 + 10x^2)$$
$$-8x^2 - 34x + 30$$
$$-(-8x^2 - 40x)$$
$$6x + 30$$
$$-(6x + 30)$$
$$0$$

b) Vet fra a) at -5 er et nullpunkt (Delelig på $x+5$)

$$2x^2 - 8x + 6 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 2 \cdot 6}}{2 \cdot 2}$$

$$x = \frac{8 \pm \sqrt{64 - 48}}{4}$$

$$x_1 = \frac{8+4}{4} = \underline{3}$$

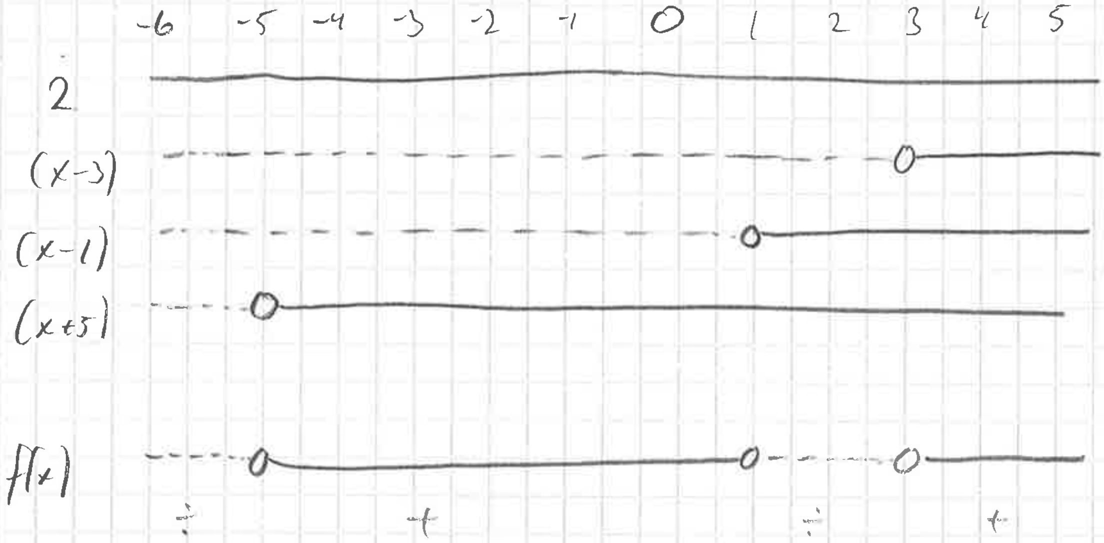
$$x_2 = \frac{8-4}{4} = \underline{1}$$

Faktorisert:

$$f(x) = \underline{\underline{2(x-3)(x-1)(x+5)}}$$

$\underline{\underline{\text{Nullpunkt: } x = -5,\ x = 3 \text{ og } x = 1}}$

c) $f(x) < 0$



$f(x) < 0 \qquad x \in \langle \leftarrow, -5 \rangle \cup \langle 1, 3 \rangle$

$x < -5 \;\wedge\; 1 < x < 3$

2. a) $g(x) = x^3 - 3x + 2 \qquad x \in \mathbb{R}$

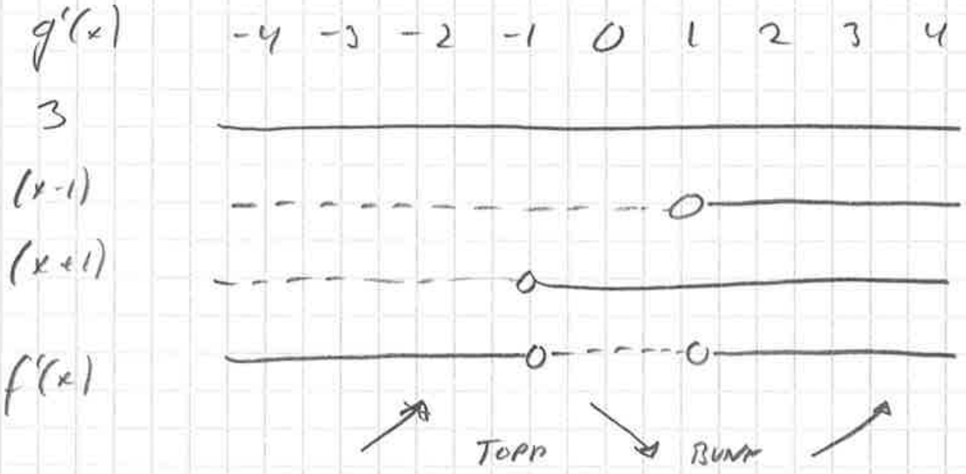
$g'(x) = 3x^2 - 3 \qquad g'(x) = 0$

$3x^2 - 3 = 0$

$3x^2 = 3$

$g'(x) = 3(x^2 - 1) \qquad x^2 = 1$

$g'(x) = 3(x-1)(x+1) \qquad x \pm 1$



TOPP: $g(-1) = (-1)^3 - 3(-1) + 2$

$= -1 + 3 + 2 = 4 \qquad \underline{\underline{\text{TOPP}(-1, 4)}}$

BUNN $\quad g(1) = 1^3 - 3(1) + 2 = 0 \qquad \underline{\underline{\text{BUNN}(1, 0)}}$

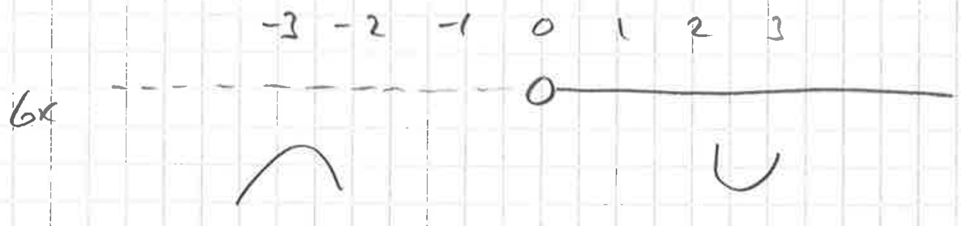
Alternativ 2. derivertest:

$g''(x) = 6x$

$g''(-1) = -6$ Negativ $\rightarrow x = -1$ er en topp

$g''(1) = 6$ Positiv $\rightarrow x = 1$ er en bunn

b) $g''(x) = 6x$



$g(x)$ er konkav for $x < 0$

$g(x)$ er konveks for $x > 0$

Vendepunkt for $x = 0$

$g(0) = 2$ Vendepunkt i $(0,2)$

c) $g'(0) = -2(0) - 3 = -3 = a$ $x = 0$ $y = 2$

$y - y_1 = a(x - x_1)$

$y - 2 = -3(x - 0)$

$\underline{\underline{y = -3x + 2}}$

5

3. a)

$$g(x) = \frac{2x}{x^2+1}$$

$$g'(x) = \frac{2(x^2+1) - 2x(2x)}{(x^2+1)^2} = \frac{2x^2+2-4x^2}{(x^2+1)^2} =$$

$$= \underline{\underline{\frac{-2x^2+2}{(x^2+1)^2}}}$$

b) $h(x) = (1+x)e^{2x}$

$$h'(x) = 1 \cdot e^{2x} + (1+x)e^{2x} \cdot 2$$

$$= e^{2x} + (e^{2x} + xe^{2x})2$$

$$= e^{2x} + 2e^{2x} + 2xe^{2x} = \underline{\underline{3e^{2x} + 2xe^{2x}}}$$

$$= \underline{\underline{e^{2x}(3+2x)}}$$

c) $f(x) = \ln(x^3 - 2x^2 + 3)$

$$f'(x) = \frac{1}{x^3 - 2x^2 + 3} \cdot 3x^2 - 4x = \underline{\underline{\frac{x(3x-4)}{x^3 - 2x^2 + 3}}}$$

4. a)

$$S(n) = n\left(a_1 + \frac{(n-1)d}{2}\right)$$

$$S(10) = 10\left(3 + \frac{(10-1)\cdot 5}{2}\right)$$

$$S(10) = 10(3 + 22{,}5)$$

$$S(10) = 10(25{,}5) = \underline{\underline{255}}$$

b)

$$S(n) = a_1 \frac{(1-k^n)}{(1-k)}$$

$$S(6) = 4{,}5 \cdot \frac{(1-1{,}5^6)}{(1-1{,}5)} = \underline{\underline{93{,}51}}$$

c)

$$a_n = a_1 \cdot k^{(n-1)}$$

$$a_5 = a_1 \cdot k^4$$

$$a_5 = 20 \cdot \left(\frac{1}{5}\right)^4 = \underline{0{,}032}$$

d)

$$S(n) = n\left(a_1 + \frac{(n-1)d}{2}\right)$$

$$100 = n\left(2 + \frac{(n-1)\cdot 3}{2}\right)$$

$$100 = 2n + \frac{(3n-3)n}{2} \quad \Big| \cdot 2$$

$$200 = 4n + 3n^2 - 3n$$

$$3n^2 + 3 - 200 = 0$$

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$n = \frac{-1 \pm \sqrt{1 - 4 \cdot 3 \cdot (-200)}}{2 \cdot 3}$$

$$n = \frac{-1 \pm \sqrt{2401}}{6} = \frac{-1 \pm 49}{6}$$

$\underline{\underline{n = 8}}$, negativ lösn. utesluten

5 a) Vinningsoptimum

$$I'(x) = K'(x)$$

$$I'(x) = -0{,}2x + 80$$

$$K'(x) = 0{,}4x + 20$$

$$-0{,}2x + 80 = 0{,}4x + 20$$

$$-0{,}6x = -60$$

$$\underline{\underline{x = 100}}$$

Maksimal profitt $= I(100) - K(100)$

$$= -0{,}1(100)^2 + 80(100) - \left(0{,}2(100)^2 + 20(100) + 2000\right)$$

$$= -1000 + 8000 - 2000 - 2000 - 2000$$

$$\underline{\underline{= 1000}}$$

b) Kostnadsoptimum: $K'(x) = E(x)$ $\qquad E(x) = \frac{K(x)}{x}$

$$E(x) = \frac{K(x)}{x} = \frac{0{,}2x^2 + 20x + 2000}{x} = 0{,}2x + 20 + \frac{2000}{x}$$

$$0{,}2x + 20 + \frac{2000}{x} = 0{,}4x + 20 \qquad | \cdot x$$

$$0{,}2x^2 + 20x + 2000 = 0{,}4x^2 + 20x$$

$$0{,}2x^2 - 0{,}4x^2 + 20x - 20x + 2000 = 0$$

$$-0{,}2x^2 = -2000$$

$$x^2 = 1000$$

$$\underline{\underline{x = 100}}$$

c) Laveste enhetskostnad

$$E(100) = 0{,}2(100) + 20 + \frac{2000}{100} = 20 + 20 + 20 = \underline{\underline{60}}$$

d) $$E_x f(x) = \frac{x}{f(x)} \cdot f'(x)$$

$x(p) = 240 - p^2$

$x'(p) = -2p$

$$E_p\, x(p) = \frac{p}{240-p^2} \cdot (-2p) = \underline{\underline{\frac{-2p^2}{240-p^2}}}$$

$$E_p\, x(10) = \frac{-2(10)^2}{240-(10)^2} = \frac{-200}{140} \approx \underline{\underline{-1{,}43}}$$

Hvis prisen reduseres m/ 1% vil etterspørsel stige m/ 1,43%

6. a)

$$K_n = K_0 (1+r)^n$$

$$22000 = 20000 (1,015)^n$$

$$1,015^n = \frac{22000}{20000}$$

$$1,015^n = 1,1 \qquad / \ln$$

$$n \cdot \ln 1,015 = \ln 1,1$$

$$n = \frac{\ln 1,1}{\ln 1,015} \approx \underline{\underline{6,40}}$$

b)

$$A = \frac{K \cdot r}{1-(1+r)^{-n}}$$

$$r = \frac{0,03}{12} = 0,0025$$

$$K = 50\,000$$

$$n = 4 \cdot 12 = 48$$

$$A = \frac{50000 \cdot 0,0025}{1-(1+0,0025)^{-48}}$$

$$A = \underline{\underline{1106,72}}$$

Total innbetaling	$= 48 \cdot 1106,72 =$	53122,38
− Lånebeløp		50000,00
= Renter totalt		3122,38

(10)

c) $K = A \cdot \dfrac{1-(1+r)^{-n}}{r}$

$n = 12$ (Et år igjen)

$r = \dfrac{0{,}03}{12} = 0{,}0025$

$A = 1106{,}7$

$K = 1106{,}7 \left[\dfrac{1-(1{,}0025)^{-12}}{0{,}0025}\right]$

$\underline{\underline{K = 13067{,}09}}$

d) Sum renten = Σ aritmetisk rekke

$d = -$ avdrag $\cdot$ renten

$d = -1041{,}7 \cdot 0{,}0025 = -2{,}60$

$a_1 = 50000 \cdot 0{,}0025 = 125$

$S(n) = n\left(a_1 + \dfrac{(n-1)d}{2}\right)$

$S(48) = 48\left(125 + \dfrac{(48-1)(-2{,}60)}{2}\right)$

$= 48(125 - 61{,}1) = \underline{\underline{3067{,}2}}$

7. a) $f(x,y) = x^3 + y^3 + 3x^2 - 3y^2 + 1$

a) $f'_x(x,y) = 3x^2 + 6x$ $\quad f''_{xx}(x,y) = 6x + 6$ $\quad f''_{xy}(x,y) = 0$

$f'_y(x,y) = 3y^2 - 6y$ $\quad f''_{yy}(x,y) = 6y - 6$

b) $f'_x(x,y) = 3x^2 + 6x = 0$ $\quad f'_y(x,y) = 3y^2 - 6y = 0$

$3y(y-2) = 0$

$3x(x+2) = 0$

$\underline{y=0}$ og $y - 2 = 0$

$\underline{x = 0}$ og $x + 2 = 0$

$\underline{y = 2}$

$\underline{x = -2}$

Stasj.punkt	$A = f''_{xx}$	$B = f''_{xy}$	$C = f''_{yy}$	$AC - B^2$	Type
0,0	6	0	-6	-36	Sadel
0, 2	6	0	6	36	Bunn
-2,0	-6	0	-6	36	Topp
-2,2	-6	0	6	-36	Sadel

8. a) $U(x,y) = 10x^{0,6}y^{0,4}$

Begrensning $\quad 20x + 30y = 300$

III $\quad 20x + 30y - 300 = 0$

$$L(x,y) = 10x^{0,6}y^{0,4} - \lambda(20x + 30y - 300)$$

I $\quad L'_x(x,y) = 6x^{-0,4}y^{0,4} - 20\lambda = 0 \implies 20\lambda = 6x^{-0,4}y^{0,4}$

II $\quad L'_y(x,y) = 4x^{0,6}y^{-0,6} - 30\lambda = 0 \implies 30\lambda = 4x^{0,6}y^{-0,6}$

5 · 2 · 3
5 · 2 · 2

$$\lambda = \lambda$$

$$\frac{6x^{-0,4}y^{0,4}}{20} = \frac{4x^{0,6}y^{-0,6}}{30} \quad / \cdot 60$$

$$18x^{-0,4}y^{0,4} = 8x^{0,6}y^{-0,6} \quad / \, x^{0,4}y^{0,6}$$

$$18x^{0}y^{1} = 8x^{1}y^{0}$$

$$18y = 8x$$

$$y = \frac{8x}{18} = \frac{4}{9}x$$

III $\quad 20x + 30 \cdot \frac{4}{9}x = 300$

$$20x + \frac{120}{9}x = 300 \quad / \cdot 9$$

$$180x + 120x = 2700$$

$$300x = 2700$$

$$\underline{\underline{x = 9}}$$

$$y = \frac{8 \cdot 9}{18} = \underline{\underline{4}}$$

13

8. b)

Maksimal nytte:

$$U(9,4) = 10 \cdot 9^{0,6} \cdot 4^{0,4} = \underline{\underline{65,07}}$$