UTSATT EKSAMEN METODE 1 - MATEMATIKK FEB 2017

Oppg. 1.

a) $f(1) = 2(1)^3 + 2(1)^2 - 34(1) + 30 = \underline{\underline{0}}$

$f(3) = 2(3)^3 + 2(3)^2 - 34(3) + 30 = \underline{\underline{0}}$

b)
$$
\begin{array}{l}
2x^3 + 2x^2 - 34x + 30 \;:\; x+5 = \underline{\underline{2x^2 - 8x + 6}} \\
\underline{-(2x^3 + 10x^2)} \\
\quad -8x^2 - 34x + 30 \\
\quad \underline{-(-8x^2 - 40x)} \\
\qquad 6x + 30 \\
\qquad \underline{-(6x + 30)} \\
\qquad\quad 0
\end{array}
$$

c) Nullpunktene til $f(x)$ er $(1,0)$ og $(3,0)$ fra a) og $(-5,0)$ fra b)

$f(x) = \underline{\underline{2(x-1)(x-3)(x+5)}}$

OPPG. 2.

a) $$f(x) = \frac{1}{3}x^3 - 2x^2 + 3x \qquad x \in \mathbb{R}$$

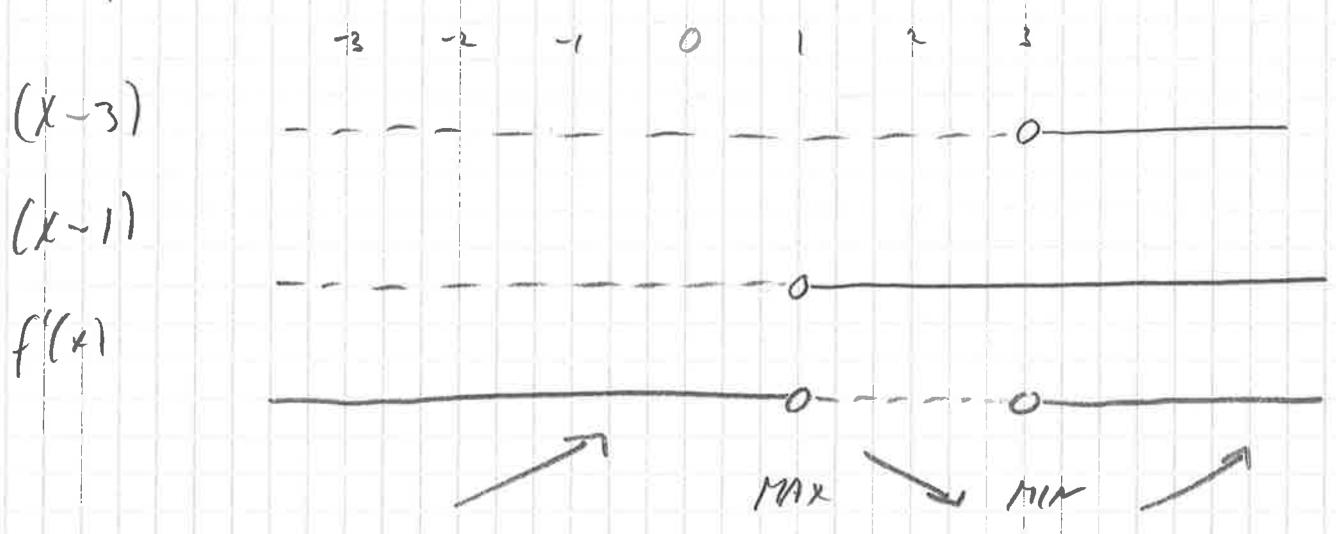
$$f'(x) = x^2 - 4x + 3$$

$$f'(x) = 0 \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1} = \frac{4 \pm \sqrt{16 - 12}}{2}$$

$$x = \frac{4 \pm 2}{2} \qquad x_1 = 3 \qquad x_2 = 1$$

$$f'(x) = (x-3)(x-1)$$



$f(x)$ vokser for $1 > x > 3$

$f(x)$ avtar for $1 < x < 3$

Maxpunkt for $x = 1$

Minpunkt for $x = 3$

b) $f''(x) = 2x - 4$

$f''(x) = 0$

$2x - 4 = 0$

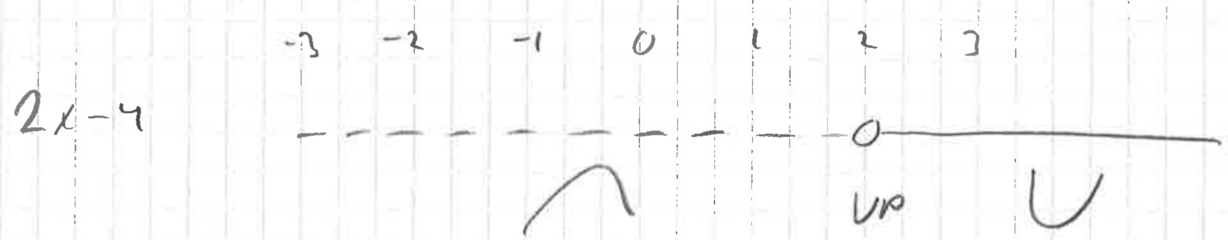
$2x = 4$

$\underline{x = 2}$

$f(2) = \frac{1}{3}(2)^3 - 2(2)^2 + 3(2)$

$= \frac{8}{3} - 8 + 6 = \frac{8}{3} - \frac{6}{3} = \underline{\underline{\frac{2}{3}}}$

$f'(2) = 2^2 - 4 \cdot 2 + 3 = \underline{-1}$



$f(x)$ e konkav $x < 2$ V.p for $x = 2$ $y = \frac{2}{3}$

$f(x)$ er konveks $x > 2$

Tangent i $(2, \frac{2}{3})$

$y - y_1 = a(x - x_1)$

$y - \frac{2}{3} = -1(x - 2)$

$y = -x + 2 + \frac{2}{3}$

$\underline{\underline{y = \frac{8}{3} - x}}$

OPPG. 3.

a) $f(x) = \dfrac{x^3}{x-1}$

$$f'(x) = \frac{3x^2(x-1) - x^3(1)}{(x-1)^2} = \frac{3x^3 - 3x^2 - x^3}{(x-1)^2}$$

$$= \underline{\underline{\frac{2x^3 - 3x^2}{(x-1)^2}}}$$

b) $g(x) = \frac{1}{2}x^2(x+4)$

$$g'(x) = x(x+4) + \frac{1}{2}x^2(1) = x^2 + 4x + \frac{1}{2}x^2 -$$

$$= \underline{\underline{\frac{3}{2}x^2 + 4x}}$$

c) $h(x) = e^{2x^2 + \ln(3x)}$

$$h'(x) = e^{2x^2 + \ln(3x)} \cdot \left(4x + \frac{1}{\cancel{3}x} \cdot \cancel{3}\right)$$

$$\underline{\underline{= e^{2x^2 + \ln(3x)}\left(4x + \frac{1}{x}\right)}}$$

Oppg. 4.

a) $a_1 = 50 \qquad a_9 = 32$

$$a_n = a_1 + (n-1) \cdot d$$

$$a_9 = a_1 + (9-1) \cdot d$$

$$32 = 50 + 8d$$

$$8d = 32 - 50$$

$$d = \underline{\underline{-2{,}25}}$$

b) $a_1 = 2 \qquad k = 2{,}25$

$$a_n = a_1 \cdot k^{n-1}$$

$$a_8 = a_1 \cdot k^{8-1} = 2 \cdot 2{,}25^7 \approx \underline{\underline{583{,}86}}$$

c) $S(n) = a_1 \frac{1-k^n}{1-k} = \frac{2(1-2{,}25^8)}{1-2{,}25} = \underline{\underline{1049{,}35}}$

OPPG. 5. $K(x) = 0{,}2x^2 + 60x + 80$

a) Grensekostnad $= K'(x) = \underline{0{,}4x + 60}$

Enhetskostnad $E(x) = \dfrac{K(x)}{x} = \dfrac{0{,}2x^2 + 60x + 80}{x} = \underline{\underline{0{,}2x + 60 + \dfrac{80}{x}}}$

b) I kostnadsoptimum : Grensekostnad = Enhetskostnad

$$E(x) = K'(x)$$

$$0{,}2x + 60 + \frac{80}{x} = 0{,}4x + 60 \quad / \cdot x$$

$$0{,}2x^2 + 60x + 80 = 0{,}4x^2 + 60x$$

$$0{,}2x^2 - 0{,}4x^2 = -80$$

$$-0{,}2x^2 = -80$$

$$x^2 = 400$$

Kostnadsoptimum : $\underline{\underline{x = 20}}$

Laveste enhetskostnad : $E(20) = 0{,}2(20) + 60 + \dfrac{80}{20}$

$$\underline{E(20) = 68}$$

c)

$x(p) = 100 - p^2$

$x'(p) = -2p$

$El_{x(p)} = \dfrac{-2p}{100 - p^2} \cdot p = \underline{\dfrac{-2p^2}{100 - p^2}}$

$El_p(6{,}00) = \dfrac{-2(6)^2}{100 - 6^2} = \dfrac{-72}{64} = \underline{-1{,}125}$

Elastisk, fordi $|-1{,}125| > 1$

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OPPG. 6.

a)

$$K_n = K_0 \cdot (1+r)^n$$

$$22000 = 2000(1+0{,}015)^n$$

$$1{,}015^n = 1{,}1$$

$$n \ln 1{,}015 = \ln 1{,}1$$

$$\underline{\underline{n = 6{,}40}}$$

b)

$$r = \frac{0{,}024}{12} = 0{,}002 \qquad K = 50000$$

$$n = 4 \cdot 12 = 48$$

$$A = \frac{50000 \cdot 0{,}002}{1-(1{,}002)^{-48}} \approx \underline{\underline{1093{,}51}}$$

c) Restgjeld = Nåverdi av gjenstående annuiteter

$$K = A\,\frac{(1-(1+r)^{-n})}{r}$$

Etter 3 år → 12 terminer igjen

r som før

$A = 1093{,}51$

$$K = \frac{1093{,}51(1-(1{,}002)^{-12})}{0{,}002}$$

$$\underline{\underline{K = 12953{,}11}}$$

OPPG. 7.

$$f(x,y) = \frac{1}{3}x^3 - \frac{1}{3}y^3 + x^2 - y^2 + 2$$

a) $f_x' = x^2 + 2x$ $\quad f_{xx}'' = 2x + 2$ $\quad f_{xy}'' = 0$

$f_y' = -y^2 - 2y$ $\quad f_{yy}'' = -2y - 2$

b) $f_x' = 0$ $\quad\quad f_y' = 0$

$x^2 + 2x = 0$ $\quad\quad -y^2 - 2y = 0$

$\underline{x = 0}$ $\quad\quad \underline{y = 0}$

$x + 2 = 0$ $\quad\quad (y+2) = 0$

$\underline{x = -2}$ $\quad\quad \underline{y = -2}$

Stasjonære punkter $\quad (0,0)\ (0,-2)\ (-2,0)\ (-2,-2)$

PUNKT	A f_{xx}''	B f_{xy}''	C f_{yy}''	$AC - B^2$	Konk.
$(0,0)$	2	0	-2	-4	Sadel
$(0,-2)$	2	0	2	4	Min
$(-2,0)$	-2	0	-2	4	Max
$(-2,-2)$	-2	0	2	-4	Sadel