LØSNINGSFORSLAG EKSAMEN NOV. 2016 ①

OPPG. 1. a) $g(2) = 2^3 - 5(2)^2 - 2(2) + 24 = 8$

$g(-2) = (-2)^3 - 5(-2)^2 - 2(-2) + 24 = 0$

$g(-2) = 0$, derfor er $g(x)$ delelig på $x-(-2) = x+2$

b) $x^3 - 5x^2 - 2x + 24 : x+2 = \underline{\underline{x^2 - 7x + 12}}$

$$\begin{array}{l} x^3 - 5x^2 - 2x + 24 \\ \underline{-(x^3 + 2x^2)} \\ \quad -7x^2 - 2x + 24 \\ \quad \underline{-(-7x^2 - 14x)} \\ \qquad 12x + 24 \\ \qquad \underline{-(12x + 24)} \\ \qquad\quad 0 \end{array}$$

c) Fra a) vet vi at $x = -2$ er et nullpunkt

$x^2 - 7x + 12 = 0$

$x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4 \cdot 1 \cdot 12}}{2 \cdot 1}$

$x = \dfrac{7 \pm \sqrt{1}}{2}$

$x_1 = \dfrac{8}{2} = \underline{4}$

$x_2 = \dfrac{6}{2} = \underline{3}$

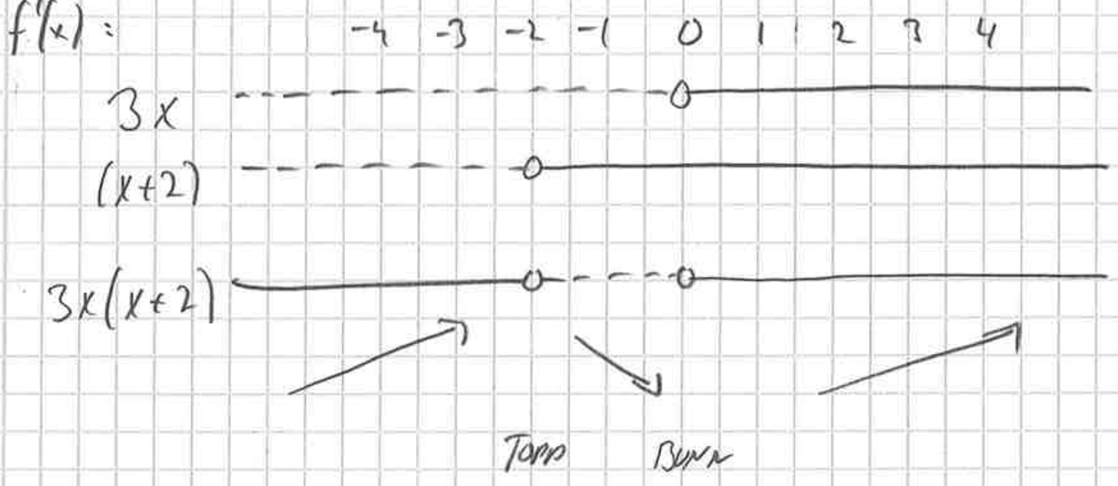
$\left.\begin{array}{l} x = -2 \\ x = 4 \\ x = 3 \end{array}\right\}$ Nullpunkter

$g(x) = \underline{\underline{(x+2)(x-4)(x-3)}}$

OPPG. 2.

a) $f(x) = x^3 + 3x^2 + 6 \qquad x \in \mathbb{R}$

$$f'(x) = 3x^2 + 6x = 3x(x+2)$$



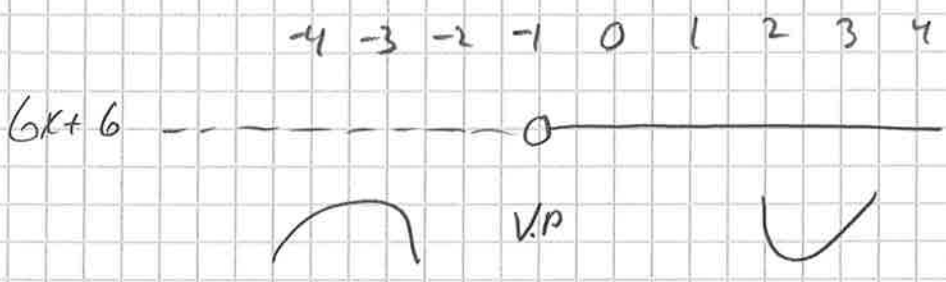
$f(x)$ vokser for $x < -2$ og $x > 0$

$f(x)$ avtar for $-2 < x < 0$

$f(x)$ har makspunkt for $x = -2$

$f(x)$ har minpunkt for $x = 0$

b) $f''(x) = 6x + 6$



$f(x)$ er konkav for $x < -1$

$f(x)$ er konveks for $x > -1$

$f(x)$ har vendepunkt for $x = -1$

$f(-1) = -1^3 + 3(-1)^2 + 6 = 8$

$f'(-1) = 3(-1)^2 + 6(-1) = -3$

$y - y_1 = a(x - x_1)$

$y - 8 = -3(x-(-1))$

$y = -3x - 3 + 8$

$\underline{\underline{y = -3x + 5}}$

OPPG. 3.

a) $f'(x) = \dfrac{(2x-7)(x-2) - (x^2-7x+12)(1)}{(x-2)^2} =$

$= \dfrac{2x^2 - 4x - 7x + 14 - x^2 + 7x - 12}{(x-2)^2}$

$= \underline{\underline{\dfrac{x^2 - 4x + 2}{(x-2)^2}}}$

b) $g'(x) = \dfrac{1}{x^4 + x^2} \cdot 4x^3 + 2x = \dfrac{2\cancel{x}(2x^2+1)}{\cancel{x}(x^3 + x)} =$

$\underline{\underline{\dfrac{2(2x^2+1)}{x^3+x}}}$

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OPPG. 3.

c) $h(x) = e^{2x^2 + \ln(3x)}$

$h'(x) = e^{2x^2 + \ln(3x)} \cdot \left(4x + \frac{1}{3x} \cdot 3\right)$

$\underline{\underline{= e^{2x^2 + \ln 3x} \cdot \left(4x + \frac{1}{x}\right)}}$

OPPG 4.

a) $a_1 = 3,5 \qquad d = 1,5$

$a_n = a_1 + (n-1) \cdot d$

$a_{25} = 3,5 + (25-1) \cdot 1,5 = \underline{\underline{39,5}}$

$S(n) = n \cdot \frac{(a_1 + a_n)}{2}$

$S(25) = \frac{25(3,5 + 39,5)}{2} = \underline{\underline{537,5}}$

b) $a_1 = 20 \qquad k = 0,2$

$a_n = a_1 \cdot k^{(n-1)}$

$a_5 = 20 \cdot 0,2^{(5-1)} = \underline{\underline{0,032}}$

$S(n) = a_1 \frac{(1 - k^n)}{(1-k)} \qquad S(8) = \frac{20(1 - 0,2^8)}{(1 - 0,2)} \approx \underline{\underline{25}}$

OPPG. 5.

a) Inntekt = pris · mengde

$$I(x) = p(x) \cdot x = (80 - 0{,}1x) \cdot x = \underline{\underline{-0{,}1x^2 + 80x}}$$

Vinningsoptimum: $I'(x) = K'(x)$

$$I'(x) = -0{,}2x + 80 \qquad\qquad -0{,}2x + 80 = 0{,}4x + 20$$

$$K'(x) = 0{,}4x + 20 \qquad\qquad -0{,}6x = -60$$

$$\underline{\underline{x = 100}}$$

Maksimal profitt:

$$I(100) - K(100) = -0{,}1(100)^2 + 80(100) - (0{,}2(100)^2 + 20(100) + 2880)$$

$$\underline{\underline{= 120}}$$

b) Enhetskostnad $= E(x) = \dfrac{K(x)}{x}$

$$E(x) = \frac{0{,}2x^2 + 20x + 2880}{x} = \underline{0{,}2x + 20 + \frac{2880}{x}}$$

Finne bunnpunkt: $E'(x) = 0$

$$0{,}2 - 2880x^{-2} = 0$$

$$0{,}2 = \frac{2880}{x^2}$$

$$x^2 = 14400 \longrightarrow \underline{\underline{x = 120}}$$

Oppg. 5 b) forts.

Laveste enhetskostnad ÷

$$E(120) = 0{,}2 \cdot 120 + 20 + \frac{2880}{120} = \underline{\underline{68}}$$

c) $x(p) = 240 - p^2$ $\qquad$ $x'(p) = -2p$

$$El_p\, x(p) = \frac{p}{x(p)} \cdot x'(p) = \frac{p}{240-p^2} \cdot -2p = \underline{\underline{\frac{-2p^2}{240-p^2}}}$$

$$El_{10} = \frac{-2(10)^2}{240-(10)^2} = \frac{-200}{140} \underline{\underline{\approx -1{,}43}}$$

Hvis prisen settes opp m/ 2% går etterspørsel ned med $2\% \cdot 1{,}43 \approx \underline{\underline{2{,}86\%}}$

OPPG 6.

$K = 300\,000$

$r = \frac{0{,}03}{2} = 0{,}015$

$n = 20 \cdot 2 = 40$

a) $$A = \frac{K \cdot r}{1 - (1+r)^{-n}}$$

$$A = \frac{300000 \cdot 0{,}015}{1 - (1{,}015)^{-40}}$$

$$\underline{\underline{A = 10028{,}13}}$$

b)

Totalt innbetalt	$10028{,}13 \cdot 40 = 401125{,}20$
− Lån	$300\,000{,}00$
= Renter	$101'125{,}2$

c) Restlån = Nåverdi av gjenværende annuiteter

$A = 10028{,}13$

$r = 0{,}015$

$n = 20$

$$K = \frac{A\left(1 - (1+r)^{-n}\right)}{r}$$

$$K = \frac{10028{,}13\left(1 - (1{,}015)^{-20}\right)}{0{,}015}$$

$$\underline{\underline{K = 172169{,}34}}$$

OPPG. 6.

d) $$1{,}08^x = 2$$

$$\ln 1{,}08^x = \ln 2$$

$$x \ln 1{,}08 = \ln 2$$

$$x = \frac{\ln 2}{\ln 1{,}08} \approx \underline{\underline{9}}$$

OPPG. 7

a) $$f(x,y) = \frac{1}{3}x^3 - \frac{1}{3}y^3 + x^2 - y^2 + 2$$

a) $f'_x = x^2 + 2x$ $\qquad f'_y = -y^2 - 2y$

$f''_{xx} = 2x + 2$ $\qquad f''_{yy} = -2y - 2$

$f''_{xy} = f''_{yx} = 0$

b) Stasjonære punkt: $f'_x = f'_y = 0$

$f'_x = 0$ $\qquad f'_y = 0$

$x^2 + 2x = 0$ $\qquad -y^2 - 2y = 0$

$x(x+2) = 0$ $\qquad y(-y-2) = 0$

$x = 0$

$x = -2$

$y = 0$

$y = -2$

Oppg. 7 b) forts.

Stasjonære punkt $(0,0)$, $(0,-2)$, $(-2,0)$, $(-2,-2)$

Punkt	$A = f''_{xx}$	$B = f''_{xy}$	$C = f''_{yy}$	$AC - B^2$	Konk.
$(0,0)$	2	0	-2	-4	Sadel
$(0,-2)$	2	0	2	4	Min
$(-2,0)$	-2	0	-2	4	Max
$(-2,-2)$	-2	0	2	-4	Sadel

Oppg. 8.

$$f(x,y) = -3x^2 + 3xy - y^2 + 6x + y$$

gitt $x + y = 11$

$$L(x) = -3x^2 + 3xy - y^2 + 6x + y - \lambda x - \lambda y + \lambda 11$$

I $\quad L'_x = -6x + 3y + 6 - \lambda = 0$

$$-6x + 3y + 6 = \lambda$$

II $\quad L'_y = 3x - 2y + 1 - \lambda = 0$

$$3x - 2y + 1 = \lambda$$

$$I = II$$
$$\lambda = \lambda$$

$$-6x + 3y + 6 = 3x - 2y + 1$$

$$-6x - 3x = -2y - 3y - 6 + 1$$

$$-9x = -5y - 5$$

$$x = \frac{5}{9}y + \frac{5}{9}$$

$$III \quad x + y = 11$$

$$\frac{5}{9}y + \frac{5}{9} + y = 11$$

$$\frac{14}{9}y = \frac{99}{9} - \frac{5}{9}$$

$$\frac{14}{9}y = \frac{94}{9}$$

$$y = \frac{94}{\cancel{9}} \cdot \frac{\cancel{9}}{14} = \frac{94}{14} = \underline{\underline{\frac{47}{7}}}$$

$$III \quad x + \frac{47}{7} = 11$$

$$x = \frac{77}{7} - \frac{47}{7} = \underline{\underline{\frac{30}{7}}}$$