Question 3 (Power Electronics):

A single-phase single-pulse diode rectifier (Fig. 3.1), with a resistive load (R=100 Ω), is fed from an AC voltage source $v_s(t) = Vs \cdot \sqrt{2} \cdot \sin(\omega t)$, where Vs=240 V, f=50 Hz.

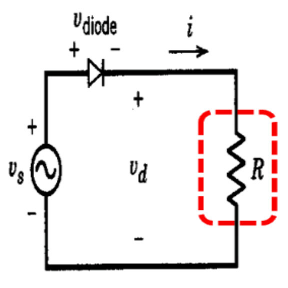


Fig. 3.1. A single-phase single-pulse diode rectifier with a resistive load

a) Calculate the average and RMS values of the current and power dissipation in the load resistor;

Solution: $v_s(t) = Vs \cdot \sqrt{2} \cdot \sin(\omega t) = V_m \cdot \sin(\omega t); V_m = peak\_value$

For this converter, the diode will conducts (becomes forward biased) whenever the supply voltage (vs) is positive.

The average value of the load voltage (Vd) can be calculated as follows:

$$V_d = \frac{1}{2 \cdot \pi} \cdot \int_0^{\pi} v_s(\omega t) \cdot d(\omega t) = \frac{V_m}{2\pi} \cdot (-\cos(\pi) + \cos(0^0)) = \frac{V_m}{\pi}$$

Since the load is resistive, therefore the load voltage and current are in phase. Consequently, the average value of the load current (Id) is:

$$I_d = \frac{V_d}{R} = \frac{V_m}{\pi \cdot R} = \frac{240 \cdot \sqrt{2}}{\pi \cdot 100} = 1.08[A]$$

The average output DC power dissipated into the load resistor is given by:

$$P_d = V_d \cdot I_d = \frac{V_d^{\,2}}{R} = I_d^{\,2} \cdot R = 1.08^2 \cdot 100 = 116.64[W]$$

The RMS value of the load voltage (Vo,rms) can be calculated as follows:

$$V_{0,rms}=\sqrt{\frac{1}{2\pi}\cdot\int_0^{\pi} v_s{}^2(\omega t)\cdot d(\omega t)}=\sqrt{\frac{(V_m)^2}{2\pi}\cdot\int_0^{\pi}\frac{1}{2}\left(1-\cos(2\omega t)\cdot d(\omega t)\right)}=\frac{V_m}{2}$$

Therefore the rms value of the load current ($I_{0,rms}$) is:

$$I_{0,rms}=\frac{V_{0,rms}}{R}=\frac{V_m}{2R}=\frac{240\cdot\sqrt{2}}{2\cdot 100}=1.7[A]$$

The output AC power (rms value) is given by:

$$P_{0,rms}=V_{0,rms}\cdot I_{0,rms}=I^2{}_{0,rms}\cdot R=1.7^2\cdot 100=289[W]$$

b) To evaluate the performance of the rectifier, calculate the efficiency of the rectification and the power factor;

Solution: The efficiency of the rectification and the PF are given by

$$\eta=\frac{P_d}{P_{0,rms}}=116.64/289=0.4$$

$$PF=\frac{P_{0,rms}}{P_{in}}=\frac{289}{240\cdot 1.7}=0.708$$

c) Calculate the load voltage ripple factor or distortion factor in terms of average and RMS values and explain the result (the value obtained);

Solution: The ripple factor can be expressed as

$$RF=\frac{\sqrt{V^2{}_{0,rms}-V^2{}_d}}{V_d}=\frac{\sqrt{I^2{}_{0,rms}-I^2{}_d}}{I_d}=\sqrt{\left(\frac{1.7}{1.08}\right)^2-1}=1.21$$

This value 1.21 for a half-wave rectifier is large, since the ideal value of the ripple factor should be zero for the output dc voltage.

d) Replace the resistive load with an inductive-resistive one (R-L load), having R=100 Ω, L=0.06 H. Sketch the load voltage and current waveforms for this case and explain the four distinct regions for each supply period (cycle);

Solution: If the load now consists of a series resistor and inductor, the current will flow through the negative cycle as well. The figure below shows the voltage and current waveforms for this case, highlighting four distinct regions for one cycle.

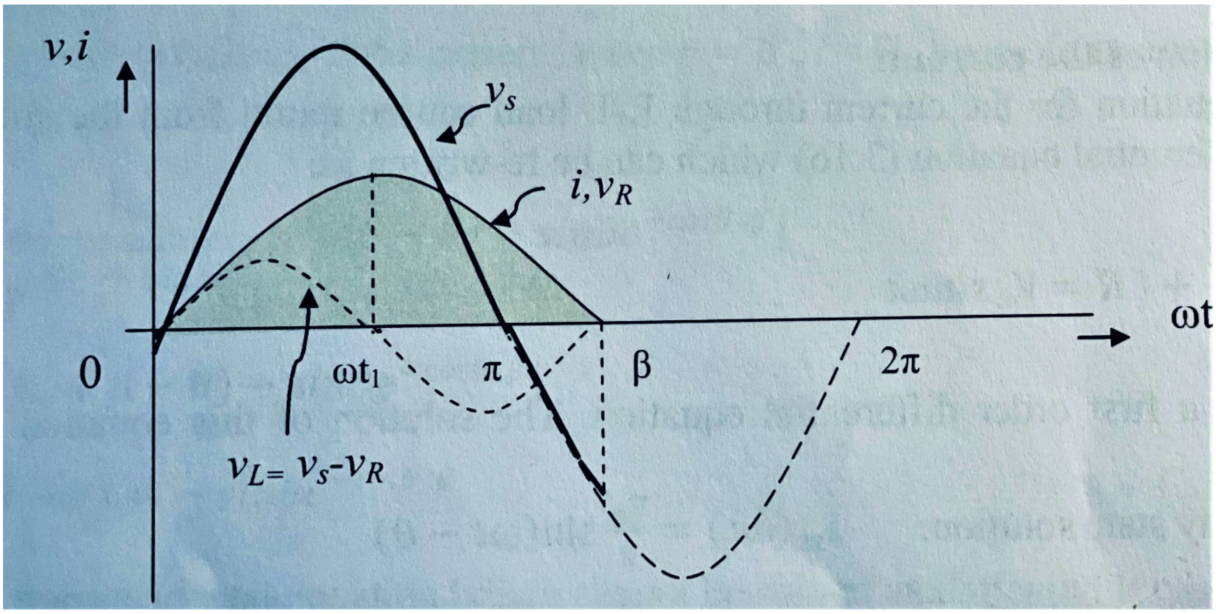


Fig. 3.2. The waveforms for single-phase single-pulse diode rectifier with a series resistive-inductive load.

During conduction period, applying KVL we have: vs=v0=vR+vL

Each period/cycle can be divided into 4 distinct regions:

1) $[0-\omega t_1]$: the current rises from zero to peak, which lags the voltage peak due to circuit inductance. In this range $v_L > 0$ and the inductance store energy
2) $[\omega t_1 - \pi]$: the current decays, and hence $v_L < 0$. Both source and inductance supply energy to R
3) $[\pi - \beta]$: the current continues to decay until it reaches zero, $v_L < 0$ and hence energy is supplied by inductance to both source and inductance
4) $[\beta - 2\pi]$: at β the current reaches zero and the diode cut-out. The current remains zero until the beginning of next positive half-cycle.

e) Calculate the average DC output voltage for the topology from the previous case (3d).

Solution: For this case, single-phase single-pulse diode rectifier with a series resistive-inductive load, the average dc voltage can be expressed as:

$$V_d = \frac{1}{2\pi} \cdot \int_0^{\beta} v_s(\omega t) \cdot d(\omega t) = \frac{V_m}{2\pi}\left[-\cos(\beta) + \cos(0)\right] = \frac{V_m}{2\pi}\left[1 - \cos(\beta)\right]$$

To calculate Vd, we need first to get the extinction angle β.

$$\beta = 180^0 + \theta; \tan(\theta) = \frac{\omega L}{R} = \frac{2\pi f \cdot L = 20}{100} = 0.2$$
$$\theta = arctg(0.2) = 11.3^0 \Rightarrow \beta = 191.3^0 = 3.34[rad]$$

Knowing β, we can calculate Vd now: $V_d = \frac{240 \cdot \sqrt{2}}{2\pi} \cdot \left[1 - \cos(3.34)\right] = 106.95[V]$

Question 4 (Power Electronics):

For a single-phase one-quadrant boost rectifier with power factor correction, the input AC voltage source Vin=220 V (RMS value), f=50 Hz and the output DC voltage is kept constant at Vd=350 V. The converter is supplying a DC motor drive (DC-DC converter + DC motor) having the following parameters: the nominal voltage, power and speed of the motor V_N=500 V, P_N=2500 W, n_N=2000 rpm.

a) Draw the entire power electronic equipment (converter) highlighting the DC-DC converter type (topology) as part of the DC motor drive;

Solution: A single-phase one-quadrant boost rectifier with power factor correction, contains a diode-bridge rectifier and a boost-converter, as can be seen in Fig. 4.1. Since the output voltage of this converter is Vd=350 V and the DC motor speed should be modified in the range of n=(0.5 ÷ 1) n_N , which means that the motor armature voltage will vary between Vo=(250-500) V, we need a **DC-DC buck-boost converter** in between to drive the motor.

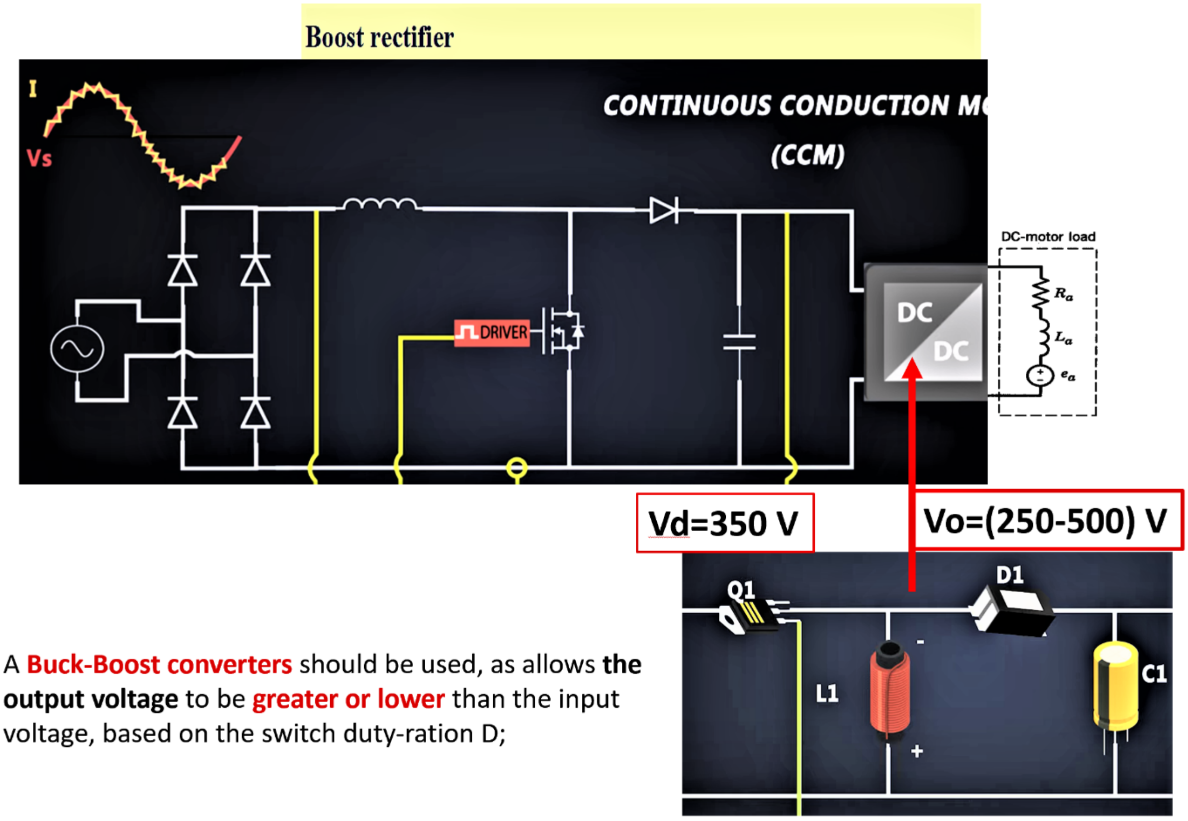


Fig. 4.1. The entire power electronic equipment (boost rectifier + DC-DC buck-boost converter + DC motor as load)

b) Design the DC-DC converter of the DC motor drive, finding the key parameters, such that the converter is able to modify the motor speed in the range $n=(0.5 \div 1)\ n_N$. Assume that the converter is working at the switching frequency fs=100 kHz, the inductor current ripple is Δi_L=10%, the voltage output ripple is Δvo=± 2%. The motor is a PM DC motor working with a constant mechanical torque over the speed range. Neglect the losses and consider the efficiency 100%.

Solution: The DC-DC converter of the DC motor drive, for which we should find the key-parameters is a buck-boost converter.

Since the converter should be able to modify the speed from 2000 rpm at V0max=500 V to 1000 rpm at Vomin=250 V, we can calculate the maximum and minimum duty ratio/cycle, Dmin and Dmax. The ratio between the input (Vd) and output (Vo) voltage can be written as:

$$\frac{V_0}{V_d} = \frac{D}{1-D} \Rightarrow D_{\min} = 0.415; D_{\max} = 0.59$$

The output current is: $I_0 = \frac{P_0}{V_0} = \frac{2500}{500} = 5[A]$

As the input voltage of the converter should be kept constant (Vd=350 V), we will take this in consideration when we calculate the inductor and capacitor of the converter.

To dimension properly the components, we need to calculate the inductance at the boundary between CCM and DCM:

$$I_{0B} = \frac{V_0}{2L_{0\min}} \cdot T_s \cdot (1-D)^2 = \frac{V_d \cdot T_s \cdot D \cdot (1-D)}{2L_{0\min}} \Rightarrow L_{0\min}$$

$$L_{0\min} = \frac{V_d \cdot D \cdot (1-D)}{2 f_s \cdot I_{0B}} = \frac{350 \cdot 0.24}{2 \cdot 10^5 \cdot 0.25} = 1.7[mH]; D = D_{\min}$$

$$where, I_{LB} = I_{0B} = \frac{\Delta i_L}{2} = 0.25$$

The output voltage ripple is $\Delta v_0 = 0.02 \cdot 500 = 10[V]$

The capacitor can be obtained from

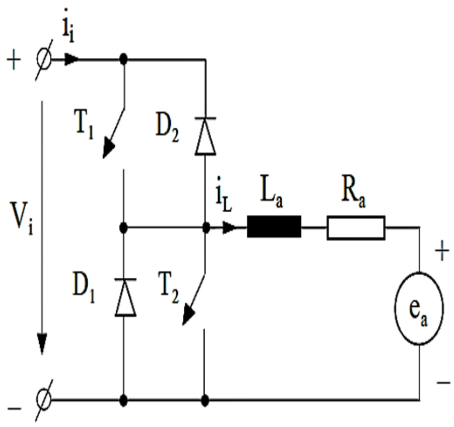
$$\Delta v_0 = \Delta v_C = \frac{V_d \cdot D^2}{C \cdot f_s \cdot R_0 \cdot (1-D)} \Rightarrow C = \frac{350 \cdot 0.415^2}{10^6 \cdot (1-0.415) \cdot 100} = 1[\mu F];$$

$$where\_R_0 = \frac{V_0}{I_0} = 100[\Omega]$$

c) Draw the entire power electronic equipment / converter such that to assure the bidirectional power flow considering the DC machine working as motor and generator as well. Explain the differences with the previous case;

Solution: To assure the bidirectional power flow, when the DC machine is working as motor and generator, the buck-boost converter should be replaced by a half-bridge or full-bridge DC-DC converter, as shown below in Fig. 4.2.

Full and half-bridge DC-DC Converters for DC-Motor Drives



Half-bridge dc-dc converter

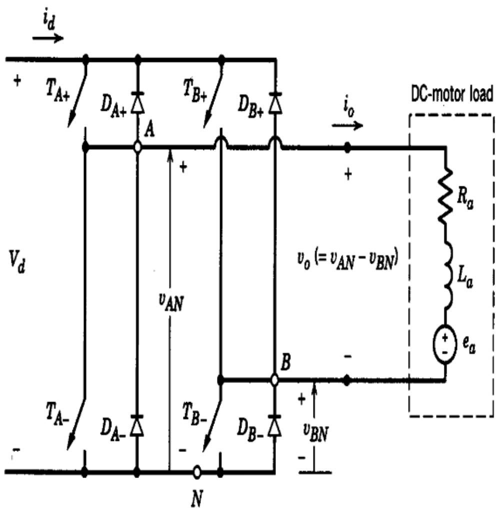


Figure 7-27 Full-bridge dc–dc converter.

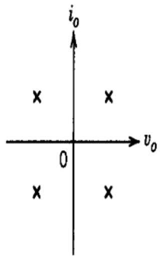
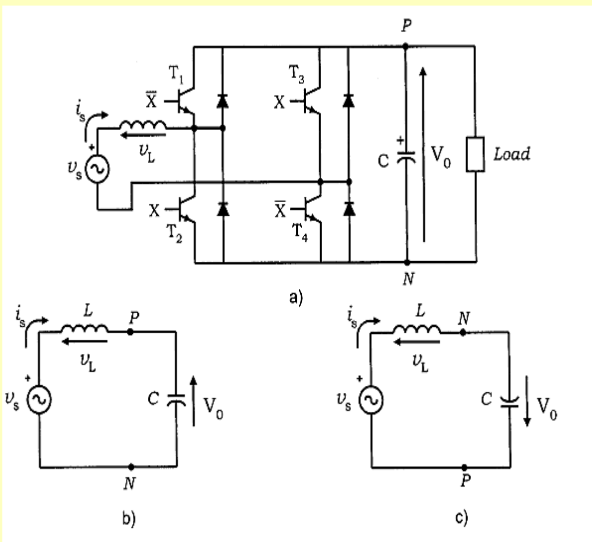


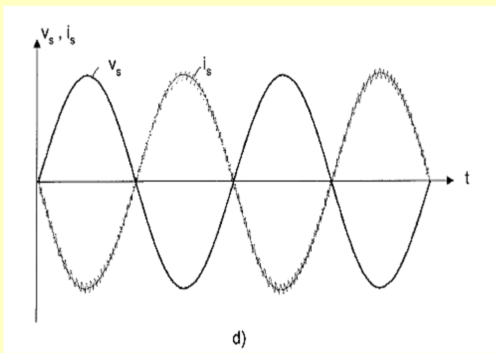
Fig. 4.2. Half-bridge and full-bridge converters for bidirectional power flow.

If the power is injected to the grid, the boost rectifier should also be replaced by a bidirectional converter or an active rectifier, as can be seen below in Fig. 4.3, otherwise a braking system in DC-link is required (a switch-on resistor in parallel with the capacitor).

3. Single-phase rectifier

Full-bridge





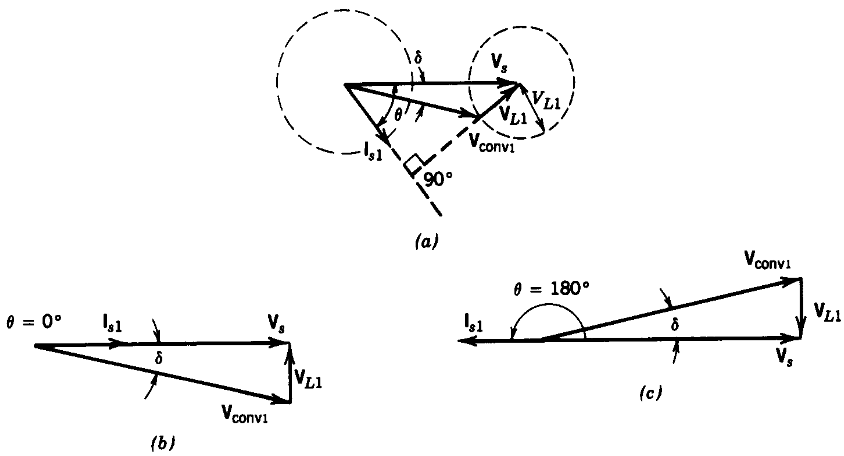


Figure 18-9 Rectification and inversion: (*a*) general phasor diagram; (*b*) rectification at unity power factor; (*c*) inversion at unity power factor.

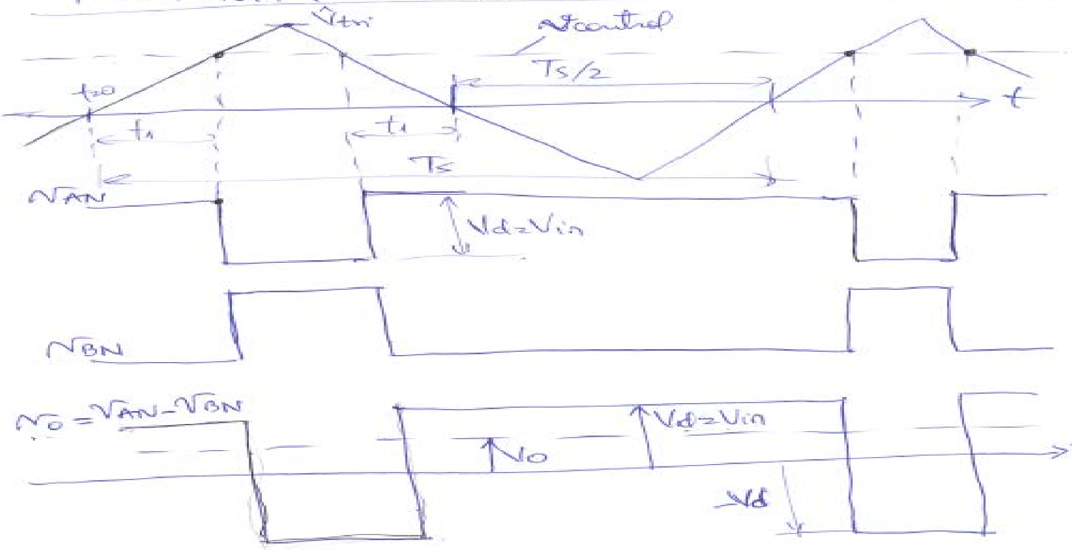
Fig. 4.3. Single-phase full-bridge bidirectional rectifier.

d) Describe the control strategy of the DC-DC converter topology used in the previous case (4c) for driving the DC machine accordingly.

Solution: The DC-DC converters used in the previous case were a half-bridge or a full-bridge bidirectional converter.

For these converters we can use a PWM (Pulse Width Modulation) technique with bipolar voltage switching (TA+, TB-) or (TA-, TB+) or a PWM technique with unipolar voltage switching (double PWM switching): switches in each leg are controlled independently of the other leg.

PWM with BIPOLAR VOLTAGE SWITCHING



$t_{on} = 2 \cdot t_1 + \frac{1}{2} T_s \; ; \; t_1 = \frac{v_{control}}{\hat{V}_{tri}} \cdot \frac{T_s}{4}$

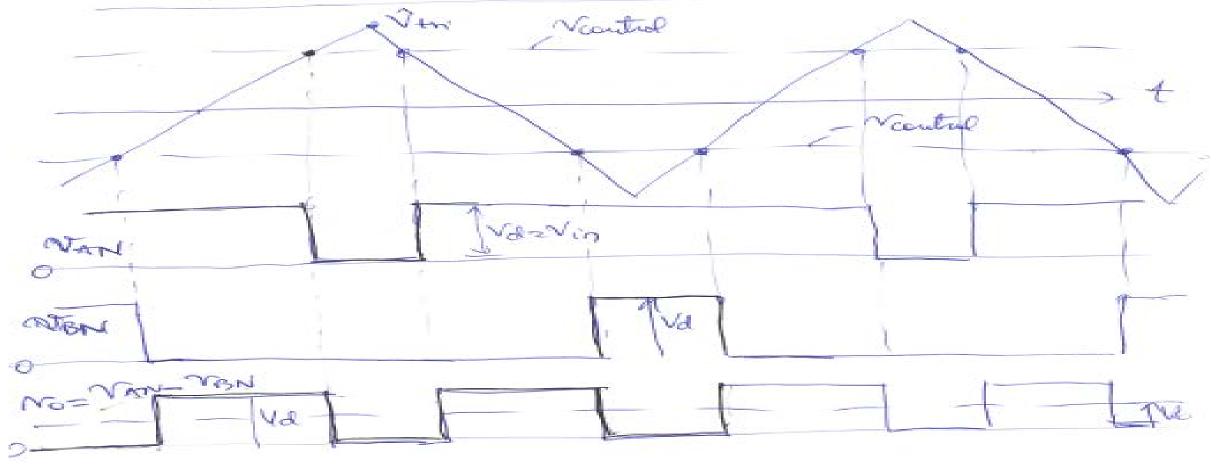
$D_1 = \frac{t_{on}}{T_s} = \frac{1}{2}\left(1 + \frac{v_{control}}{\hat{V}_{tri}}\right) \; ; (T_{A+}, T_{B-})$

$D_2 = 1 - D_1 \; ; (T_{B+}, T_{A-})$

$V_o = V_{AN} - V_{BN} = D_1 \cdot V_d - D_2 V_d = (2 D_1 - 1) \cdot V_d$

$V_o = \frac{V_d}{\hat{V}_{tri}} \cdot v_{control} = K \cdot v_{control}$

PWM with UNIPOLAR VOLTAGE SWITCHING



$T_{A+} = ON$ if $v_{control} > v_{tri}$

&

$T_{B+} = ON$ if $-v_{control} > v_{tri}$

$D_1 = \frac{1}{2}\left(\frac{v_{control}}{\hat{V}_{tri}} + 1\right) \; ; T_{A+}$

$D_2 = 1 - D_1 \; ; T_{B+}$

$V_o = (2 D_1 - 1) \cdot V_d = \frac{V_d}{\hat{V}_{tri}} v_{control}$