Høgskolen i Østfold

ASSESSMENT GUIDELINES

Course code:	**IRE35017**
Course name:	**Power Electronics and relay protection**
Form of examination:	**Written**
Date:	**11.12.2020**
Lecturer(s):	**Lucian Mihet, Kamil Dursun**
Comments:	

Please find enclosed the following documents:

Course description: https://www.hiof.no/studier/emner/ir/2020/host/ire35017.html

Exam

Formula list: https://hiof.instructure.com/courses/1416/files/388199/download?wrap=1

Solution to exam questions

Note also the following:

It is important that the student has understood the philosophy of solving the question. If the thinking and the formulae are correct, the student should not lose more than 5-10% of the points for the relevant question if only the numerical values are wrong.

Do not consider propagating errors - i.e. if the student could not find the right numerical solution at one partial question and needs the value at the next partial question, he should not lose any point at the next question due to the numerical error.

The student should give the answers in English. If the student writes some terms in Norwegian due to language problems this will be accepted. The quality of the language, grammar etc. will not affect the grade.

Due to the corona pandemic, this exam was carried out as home exam. The only forbidden aid is collaboration between students. It is impossible to control the students at home so it is important that the evaluators check the answers carefully to find out about cheating.

Question 1 (Relay protection)

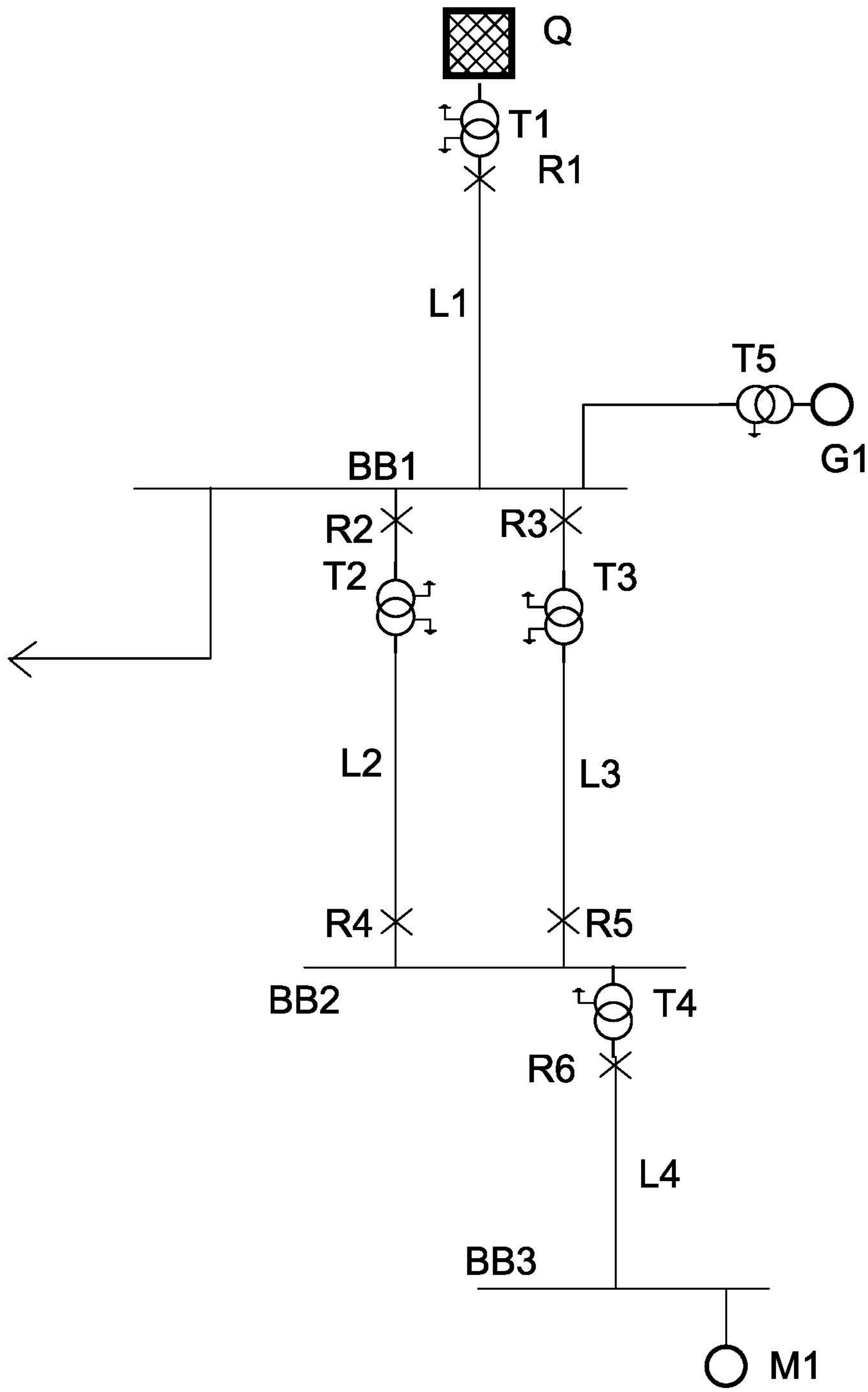

Fig. 1

Consider the circuit at Fig. 1

Q	UN=220 kV; S''= 8000 MVA; cos(Ø)=0.03; Z1 = Z2; Z0 = 3 * Z1
T1	220/132 kV; 100 MVA; uk = 10%; er = 0.95% Z0 = 0.8 * Z1; YNyn0 connection
T2	132/22 kV; 60 MVA; uk = 10%; er = 1 % Z0 = 0.8 * Z1; YNyn0 connection
T3	Same as T2
T4	22/6 kV; 15 MVA; uk=8%; er=0.9%; Z0 = 1.1 * Z1; YNd connection
T5	15 / 132 kV; 60 MVA; uk=9%; er=1.8% Z0 = 0.8 * Z1 at 132 kV side. Δ-YN connection (Δ at the 15 kV side)
G1	15 kV; 60 MVA; x_d'' = 14%
L1	30 km; Z1 = Z2 = (0.8 + j6.1) Ω/km; Z0 = (1 + j12) Ω/km
L2	40 km; Z1 = Z2 = (0.004 + j0.045) Ω/km; Z0 = (0.2 + j1.2) Ω/km
L3	Same as L2
L4	15 km; Z1 = Z2 = (0.003 + j0.032) Ω/km; Z0 = (0.01 + j0.12) Ω/km
M1	6 kV; 8 MW; cos(Ø) = 0.9; I_s/I_N = 5

a) We have a fault at BB2. Set up plus, minus and zero sequence networks. Calculate Z1, Z2 and Z0 at this point. Calculate I''_{k3}, I''_{k1}, I_S.
b) How would you set up I> and I>> for R3 and R6 to have a good discrimination? Assume definite time relays. Δt=0.3s and Pick-up/drop-off ratio 0.95. Make a drawing of the currents referred to the voltage level of BB2 since you prpbably have calculated the impedances at this level. You can later refer the currents to their relevant voltage levels.
c) How would you deal with inrush currents when connecting T2 and T3? What should you consider when setting up R2 and R3?
d) What kind of relay should you use at R4 and R5?
e) If you want that the relays should disconnect the transformers without time delay in case the fault is inside the transformers, what kind of relay can you use in addition to the overcurrent relays?

Question 2

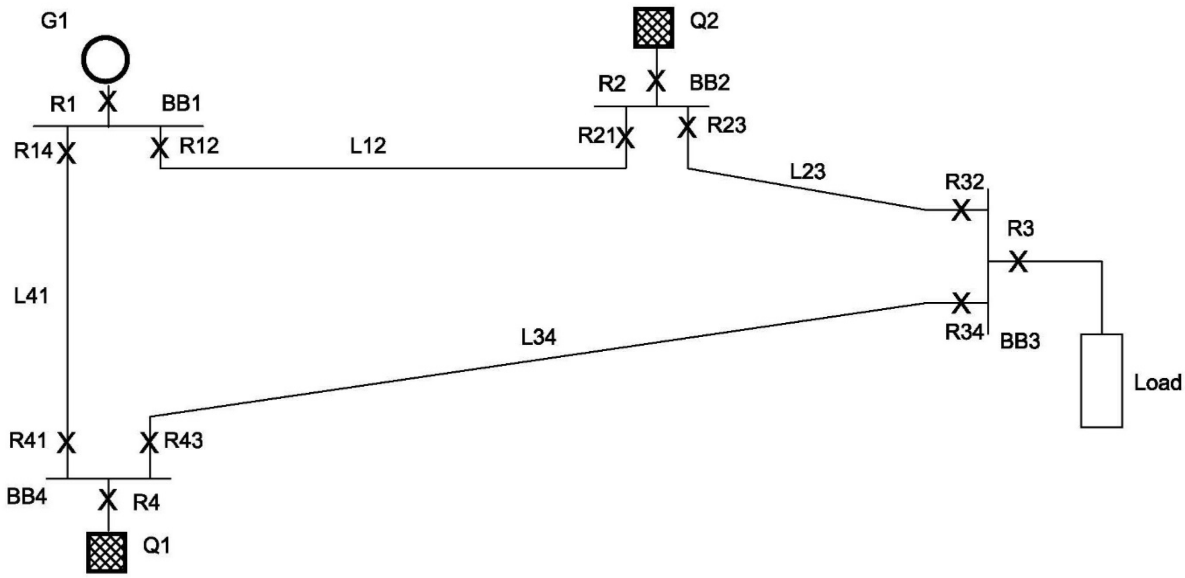

Fig. 2

On Fig.2, you see a high voltage system. We have four busbars BB1 - BB4, two infeed Q1 - Q2, one generator G1 and a load connected to BB3.

R1, R2, R3, R4 are overcurrent relays. All other relays are distance relays.

The values are:

Line	Length, impedance (all lines have same impedance per km)
L12	200 km, Z=0.03+j0.42 Ω/km
L23	150 km, Z=0.03+j0.42 Ω/km
L34	300 km, Z=0.03+j0.42 Ω/km
L41	200 km, Z=0.03+j0.42 Ω/km

a) We have a fault on L12, 30 km. from BB1. Make a table of how each distance relay will pick up the fault (Z1, Z2 or Z3) and how they will react. Which relays will trip, when and which relays will reset.
b) How would the relays react if R12is out of order? Take also into consideration the overcurrent relays.
c) It is desirable that the relays R12 and R21 react as quickly as possible (assume both are working properly). How would you set up a PUTT scheme?

Question 1 (Relay protection)

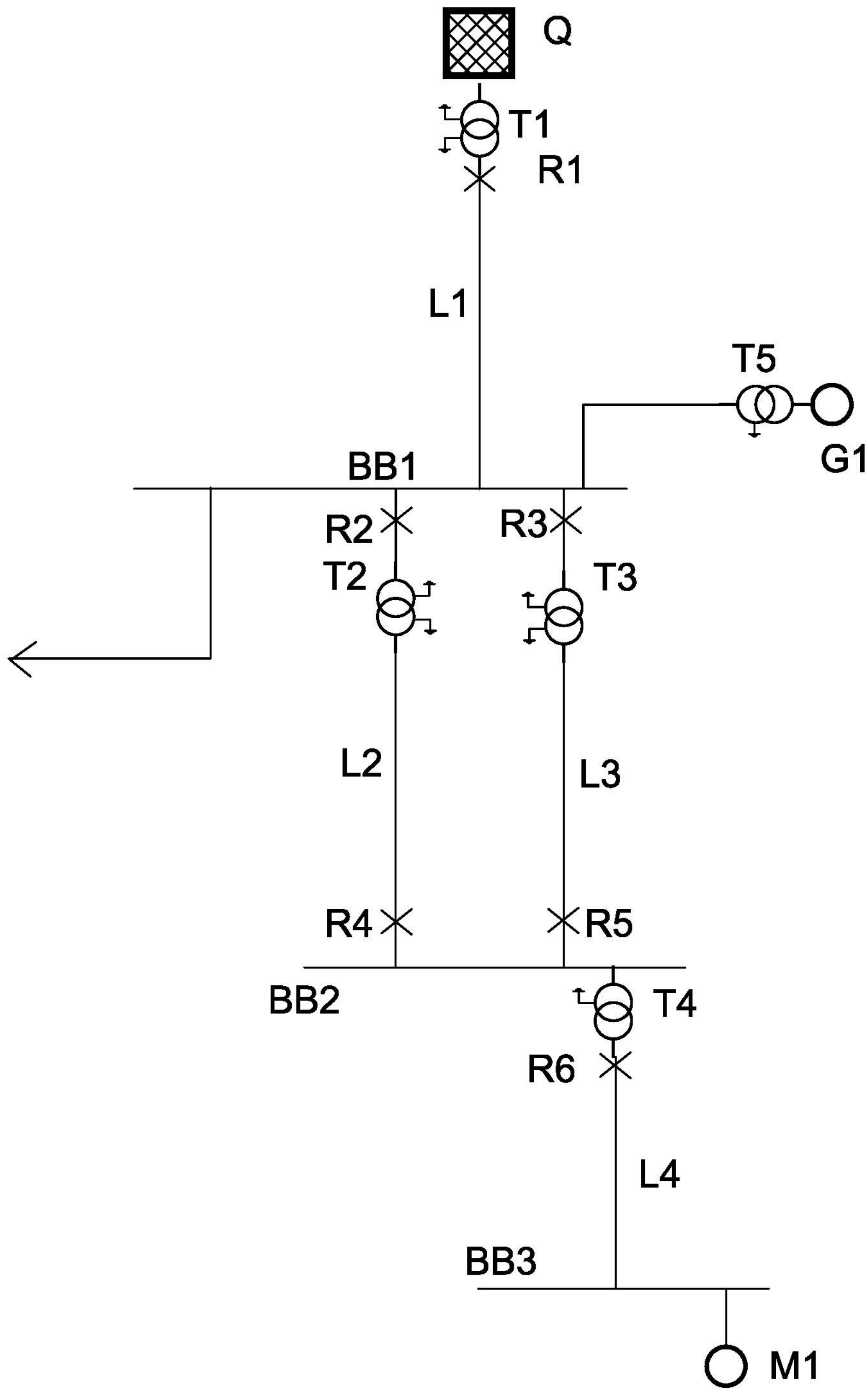

Fig. 1

Consider the circuit at Fig. 1

Q	UN=220 kV; S''= 8300 MVA; cos(Ø)=0.035; Z1 = Z2; Z0 = 3 * Z1
T1	220/132 kV; 100 MVA; uk = 11%; er = 0.95% Z0 = 0.9 * Z1; YNyn0 connection
T2	132/22 kV; 60 MVA; uk = 10%; er = 1 % Z0 = 0.8 * Z1; YNyn0 connection
T3	Same as T2
T4	22/6 kV; 20 MVA; uk=8%; er=0.9%; Z0 = 1.1 * Z1; YNd connection
T5	15 / 132 kV; 50 MVA; uk=10%; er=1.8% Z0 = 0.8 * Z1 at 132 kV side. Δ-YN connection (Δ at the 15 kV side)
G1	15 kV; 50 MVA; x_d'' = 14%
L1	35 km; Z1 = Z2 = (0.8 + j6.5) Ω/km; Z0 = (1 + j12) Ω/km
L2	40 km; Z1 = Z2 = (0.0045 + j0.04) Ω/km; Z0 = (0.2 + j1.2) Ω/km
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c) How would you deal with inrush currents when connecting T2 and T3? What should you consider when setting up R2 and R3?
d) What kind of relay should you use at R4 and R5?
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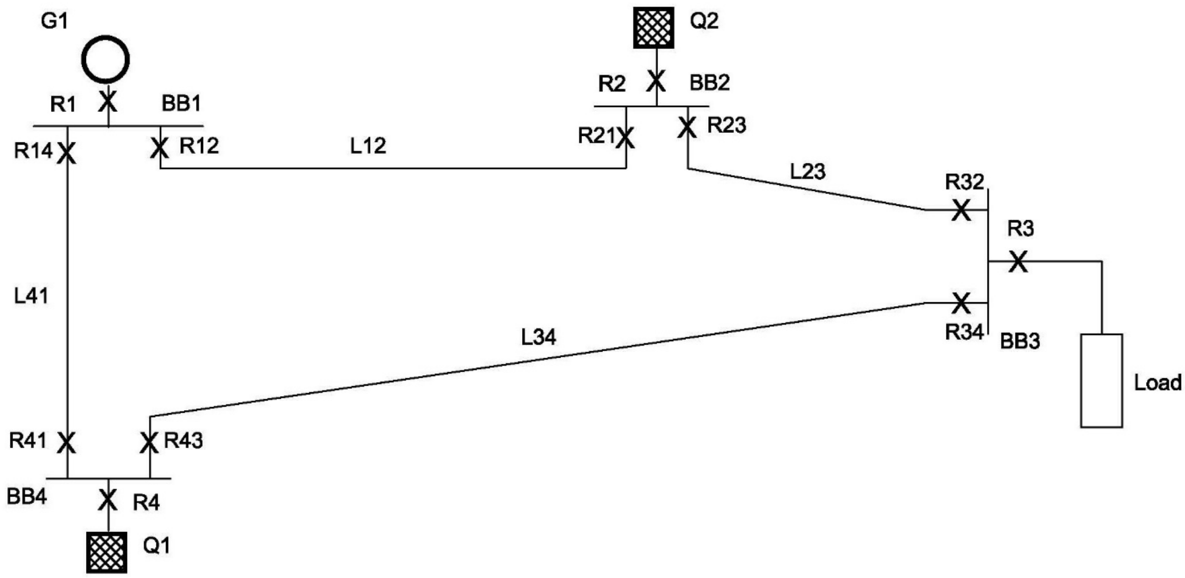
Question 2



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L34	300 km, Z=0.03+j0.42 Ω/km
L41	200 km, Z=0.03+j0.42 Ω/km

a) We have a fault on L41, 30 km. from BB4. Make a table of how each distance relay will pick up the fault (Z1, Z2 or Z3) and how they will react. Which relays will trip, when and which relays will reset.
b) How would the relays react if R41is out of order? Take also into consideration the overcurrent relays.
c) It is desirable that the relays R14 and R41 react as quickly as possible (assume both are working properly). How would you set up a POTT scheme?

Power Electronics, December 2020

Exercise 3.1. For a single-phase single-pulse thyristor rectifier with resistive load (R=15 Ohms), shown in Fig. 3.1, fed from an AC source with the voltage $v_s(t) = 220 \cdot \sin(314 \cdot t)$.

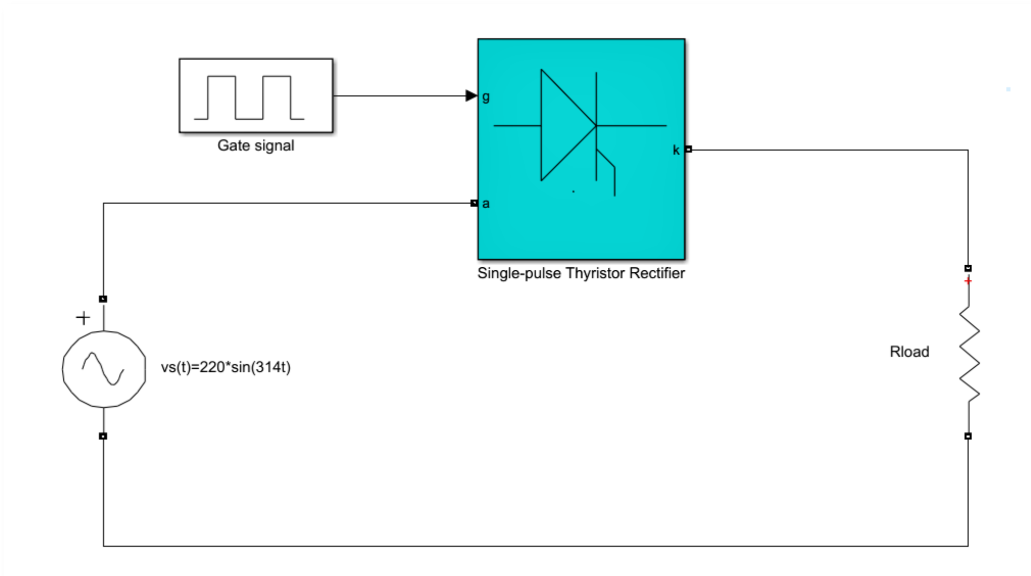


Fig. 3.1. Single-phase single-pulse thyristor rectifier

a) Explain when the average output voltage becomes maximum and when the RMS output voltage becomes zero.
b) Assuming that the average output voltage is 70% of the maximum possible output voltage, calculate the firing angle (delay angle) and the efficiency of rectification ratio.
c) Replace the single-pulse thyristor rectifier with a full-bridge thyristor rectifier and calculate the efficiency of rectification for this case/topology in the same conditions.
d) Replacing the resistive load with a DC motor represented by an equivalent circuit (Fig. 3.2) and considering the load current constant and ripple-free, draw the output voltage and the input current (Is), indicating in the sketch when the thyristors are in conduction.

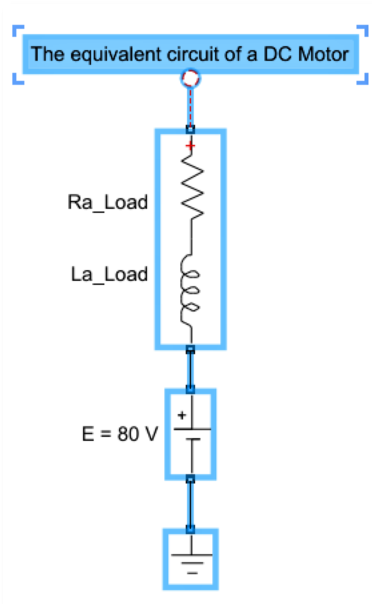


Fig. 3.2. The equivalent circuit of a DC motor.

e) For the previous case, assuming that the speed is 1000 rpm, the armature resistance Ra=2 Ohms, the induced/back emf voltage of the motor E=80 V and the armature current is kept constant at Ia=10 A, find the firing angle for this case and for the case when the speed is 500 rpm.

Exercise 4.1. A Hybrid Electric Vehicle (HEV) required a 20 kW half-bridge bidirectional converter (Fig. 4.1) to generate a 500 V from 200 V battery at a switching frequency fs=10 kHz.

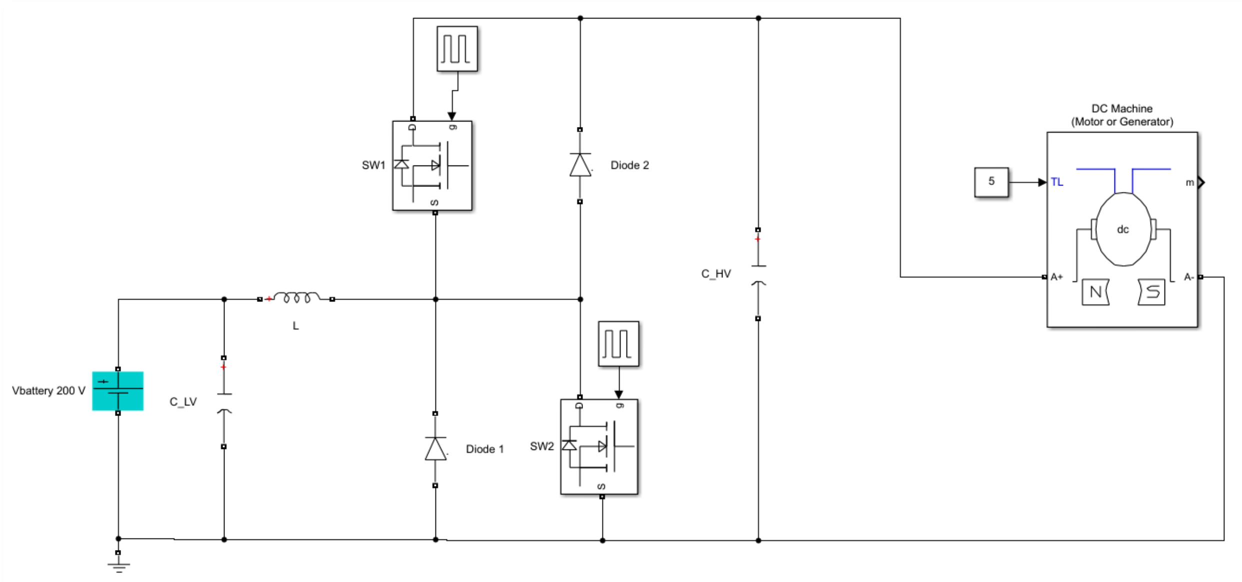


Fig. 4.1. DC machine drives by a half-bridge bidirectional converter.

a) Assuming that the electrical machine of HEV is working as generator to charge the low-voltage battery from the high-voltage DC-link, explain how the half-bridge converter works in this case and redraw the circuit such that to highlight the DC-DC converter type used for this case.
b) For the DC-DC converter type, compatible with the generating operation mode, determine the components (L and C_{LV}) considering the inductor current ripple (28%) and the voltage ripple (0.5 %), assuming ideal components and ignoring the power loss. Assume that the converter is working in continuous conduction mode (CCM) and the converter sees the battery as a load (20 kW, 200 V).
c) Calculate the input current and the minimum and maximum inductor current for CCM
d) Determine the power level at which the converter enters BCM (at the boundary between CCM and DCM)
e) As the load current and power is reduced, the converter works now in DCM (discontinuous conduction mode) for the given voltage conditions (Vemf=500 V, $V_{battery}$=200 V). Calculate the input and output currents and the inductor ripple current assuming the load power 2 kW for this case.

Power Electronics, December 2020

Exercise 3.2. For a single-phase single-pulse thyristor rectifier with resistive load (R=5 Ohms), shown in Fig. 3.1, fed from an AC source with the voltage $v_s(t) = 220 \cdot \sin(314 \cdot t)$.

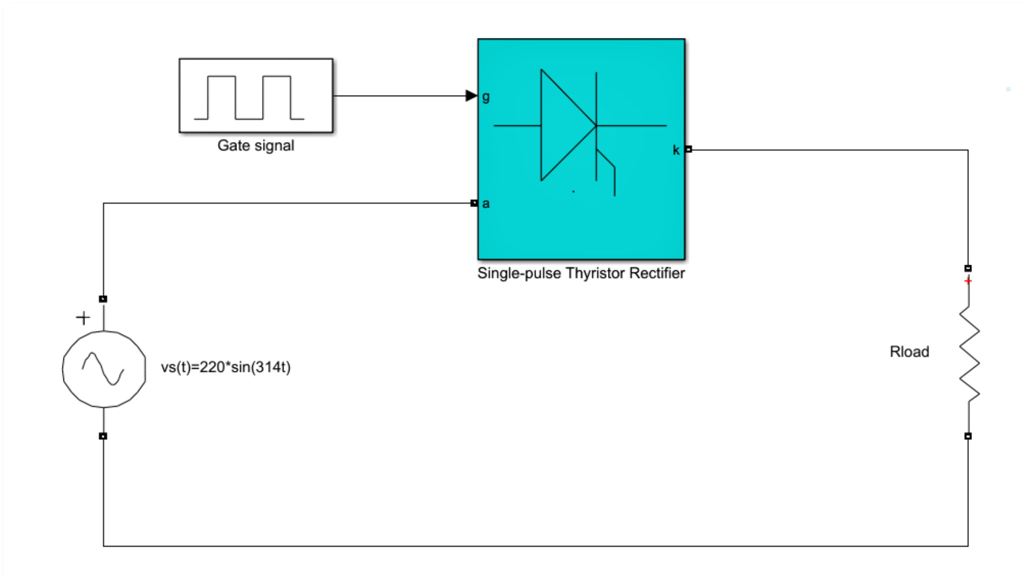


Fig. 3.1. Single-phase single-pulse thyristor rectifier

a) Explain when the average output voltage becomes maximum and when the RMS output voltage becomes zero.
b) Assuming that the average output voltage is 80% of the maximum possible output voltage, calculate the firing angle (delay angle) and the efficiency of rectification ratio.
c) Replace the single-pulse thyristor rectifier with a full-bridge thyristor rectifier and calculate the efficiency of rectification for this case/topology in the same conditions.
d) Replacing the resistive load with a DC motor represented by an equivalent circuit (Fig. 3.2) and considering the load current constant and ripple-free, draw the output voltage and the input current (Is), indicating in the sketch when the thyristors are in conduction.

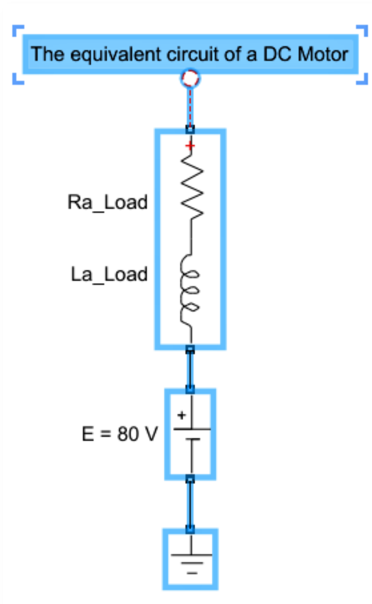


Fig. 3.2. The equivalent circuit of a DC motor.

e) For the previous case, assuming that the speed is 1000 rpm, the armature resistance Ra=2 Ohms, the induced/back emf voltage of the motor E=80 V and the armature current is kept constant at Ia=10 A, find the firing angle for this case and for the case when the speed is 500 rpm.

Exercise 4.2. A Hybrid Electric Vehicle (HEV) required a 20 kW half-bridge bidirectional converter (Fig. 4.1) to generate a 500 V from 200 V battery at a switching frequency fs=10 kHz.

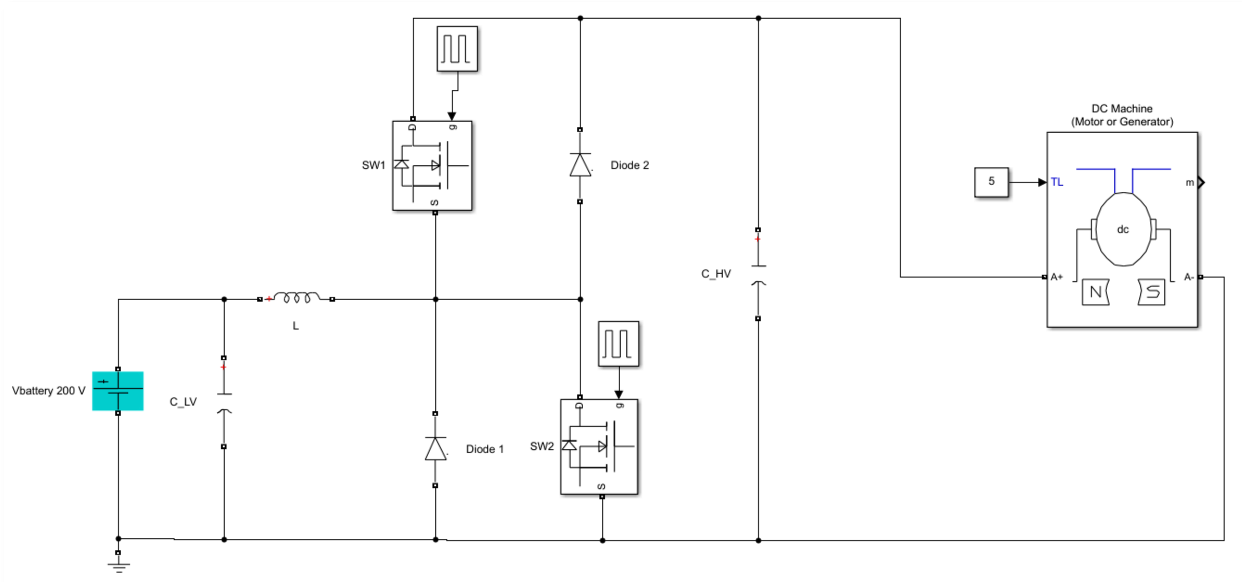


Fig. 4.1. DC machine drives by a half-bridge bidirectional converter.

a) Assuming that the DC machine of HEV is working as motor as it enables to discharge the low-voltage battery to a higher voltage DC-link, explain how the half-bridge converter works in this case and redraw the circuit such that to highlight the DC-DC converter type used for this case.
b) For the DC-DC converter type, compatible with the motoring operation mode, determine the components (L and C_{HV}) considering the inductor current ripple (28%) and the voltage ripple (0.5 %), assuming ideal components and ignoring the power loss. Assume that the converter is working in continuous conduction mode (CCM) and the converter sees the motor as a resistive load (20 kW, 500 V).
c) Calculate the input current and the minimum and maximum inductor current for CCM
d) Determine the power level at which the converter enters BCM (at the boundary between CCM and DCM)
e) As the load current and power is reduced, the converter works now in DCM (discontinuous conduction mode) for the given voltage conditions (Vemf=500 V). Calculate the input and output currents and the inductor ripple current assuming the load power 2 kW for this case.

Exam Relay Protection 2020-12-11

Solution

Question 1: We must calculate all the impedances to the voltage level at BUS2 which is 22 kV.

$$|Z_{1Q}| = 1.1 \times \frac{U^2}{S_Q} = 1.1 \times \frac{22^2}{8000} = 0{,}0666$$

$$R_{1Q} = 0.03 \times |Z_{1Q}| = 0{,}002$$

$$X_{1Q} = \sqrt{|Z_{1Q}|^2 - R_{1Q}^2} = 0{,}0665$$

$$Z_{1Q} = 0{,}002 + j\,0{,}0665 \ \Omega = Z_{2Q}$$

$$Z_{0Q} = 3 \times Z_{1Q} = 0{,}0060 + j\,0{,}1996 \ \Omega$$

ref. 22 kV.

$$|Z_{1T1}| = 0.1 \times \frac{22^2}{100} = 0{,}4840.$$

$$R_{1T1} = 0{,}0095 \times \frac{22^2}{100} = 0{,}046$$

$$X_{1T1} = \sqrt{|Z_{1T1}|^2 - R_{1T1}^2} = 0{,}4818$$

$$Z_{1T1} = 0{,}046 + j\,0{,}4818 = Z_{2T1}$$

$$Z_{0T1} = 0{,}8 \times Z_{1T1} = 0{,}0368 + j\,0{,}3854$$

$$|Z_{1T2}| = 0.1 \times \frac{22^2}{60} = 0{,}8067$$

$$R_{1T2} = 0{,}01 \times \frac{22^2}{60} = 0{,}0807$$

$$X_{1T2} = \sqrt{0{,}8067^2 - 0{,}0807^2} = 0{,}8026$$

$$Z_{1T2} = 0{,}0807 + j\,0{,}8026 = Z_{2T2}$$

$$Z_{0T2} = 0{,}8 \times Z_{1T2} = 0{,}0645 + j\,0{,}6421$$

T_3 is same as T_2

$$|Z_{1T4}| = 0{,}08 \times \frac{22^2}{15} = 2{,}5813$$

$$R_{1T4} = 0{,}009 \times \frac{22^2}{15} = 0{,}2904$$

$$X_{1T4} = \sqrt{2{,}5813^2 - 0{,}2904^2} = 2{,}5649$$

$$Z_{1T4} = 0{,}2904 + j\,2{,}5649$$

$$Z_{0T4} = 1.1 \times Z_{1T4} = 0{,}3194 + j\,2{,}8214$$

$$|Z_{1T5}| = 0{,}09 \times \frac{22^2}{60} = 0{,}7260$$

$$R_{1T5} = 0{,}018 \times \frac{22^2}{60} = 0{,}1452$$

$$X_{1T5} = \sqrt{0{,}7260^2 - 0{,}1452^2} = 0{,}7113$$

$$Z_{1T5} = 0{,}1452 + j\,0{,}7113$$

$$Z_{0T5} = 0{,}8 \times Z_{1T5} = 0{,}1162 + j\,0{,}5691$$

$$|Z_{1G1}| = 0.14 \times \frac{22^2}{60} = 1.1293$$

We do not have R_{1G1} and assume it is zero.

$$Z_{1G1} = 0 + j\,1.1293$$

$$S_M = \frac{8}{0.9} = 8.89 \text{ MVA}$$

$$Z_{1M} = \frac{22^2}{8.89 \times 5} = 0 + j\,10.89$$

$$Z_{1L1} = 30 \times (0.8 + j\,6.1) \times \left(\frac{22}{132}\right)^2$$

$$= 0.6667 + j\,5.0833$$

$$Z_{0L1} = 30 \times (1 + j\,20) \times \left(\frac{22}{132}\right)^2$$

$$Z_{0L1} = 0.8333 + j\,16.667$$

$$Z_{1L2} = 40 \times (0.003 + j\,0.032)$$

$$= 0.12 + j\,1.28$$

$$Z_{0L2} = 40 \times (0.5 + j\,2.2) = 20 + j\,88$$

L_3 same as L_2

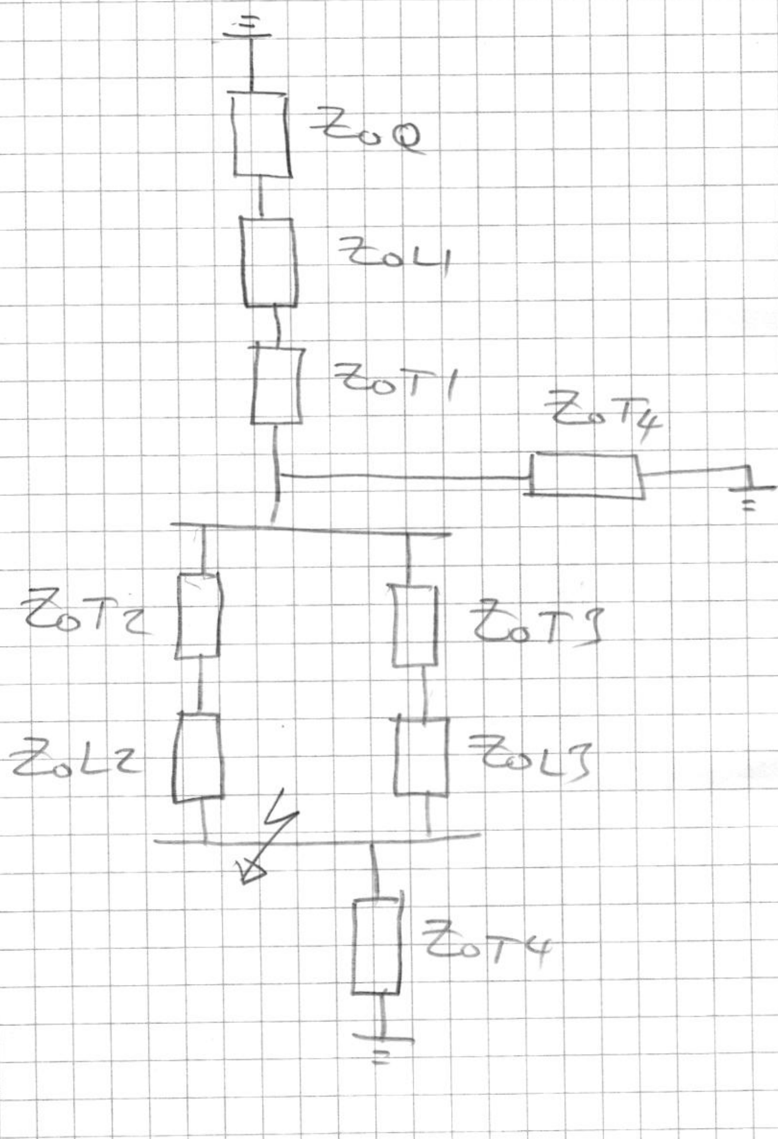
$$Z_{1L4} = 15 \times (0{,}004 + j\,0{,}045) \times \left(\frac{22}{6}\right)^2$$

$$= 0{,}8067 + j\,9{,}075$$

$$Z_1 = \left[(Z_{1Q} + Z_{1L1} + Z_{1T1}) // (Z_{1G1} + Z_{1T5}) + (Z_{1T2} + Z_{1L2})/2 \right] // (Z_{1T4} + Z_{1L4} + Z_{1M})$$

$$Z_1 = Z_2 = 0.1946 + j\,2.1931$$

To calculate Z_0, we use the following circuit:



$$Z_0 = \left[(Z_{0Q} + Z_{0T1} + Z_{0L1}) \,//\, Z_{0T5} + (Z_{0T2} + Z_{0L2})/2 \right] // Z_{0T4}$$

$$Z_0 = 0{,}3179 + j\,2.656.$$

$$I''_{k3} = \left| \frac{1.1 \times 22 \times 10^3}{\sqrt{3} \times Z_1} \right| = 6{,}35 \text{ kA}$$

$$I''_{k1} = \frac{\sqrt{3} \times 1.1 \times 22 \times 10^3}{|Z_1 + Z_2 + Z_3|} = 5{,}92 \text{ kA}$$

To calculate I_s, we need K which we get from the graph.

$$\frac{R_1}{X_1} = \frac{0{,}1946}{2{,}1931} = 0{,}09 \Rightarrow K = 1{,}7$$

$$I_s = \sqrt{2} \cdot 1.7 \times 6{,}35 \text{ kA} = 15.3 \text{ kA}.$$

(b) For relay R3

$$I_{>3} = \frac{1.2 \times S_{T2}}{0{,}95 \times \sqrt{3} \times U_{BB2}} = 1990\ A$$

To find $I_{\gg 3}$, we calculate the short circuit current at 80% of L3.

$$Z_{80\%\,L3} = \left(Z_{1Q} + Z_{1T1} + Z_{1L1} \right) // \left(Z_{1G1} + Z_{1T5} \right)$$

$$+ Z_{1T3} + 0{,}8 \times Z_{1L3}$$

$$= 0{,}3024 + j\ 3{,}2145$$

$$I_{\gg 3} = \frac{1.1 \times 22}{\sqrt{3} \times Z_{80\%\,L3}} = 4330\ A$$

For relay R6

$$I_{>6} = \frac{1.2 \times S_{T4}}{0{,}95 \times \sqrt{3}\ U_{BB2}} = 497\ A \approx 500\ A$$

Impedance at 80% of L4

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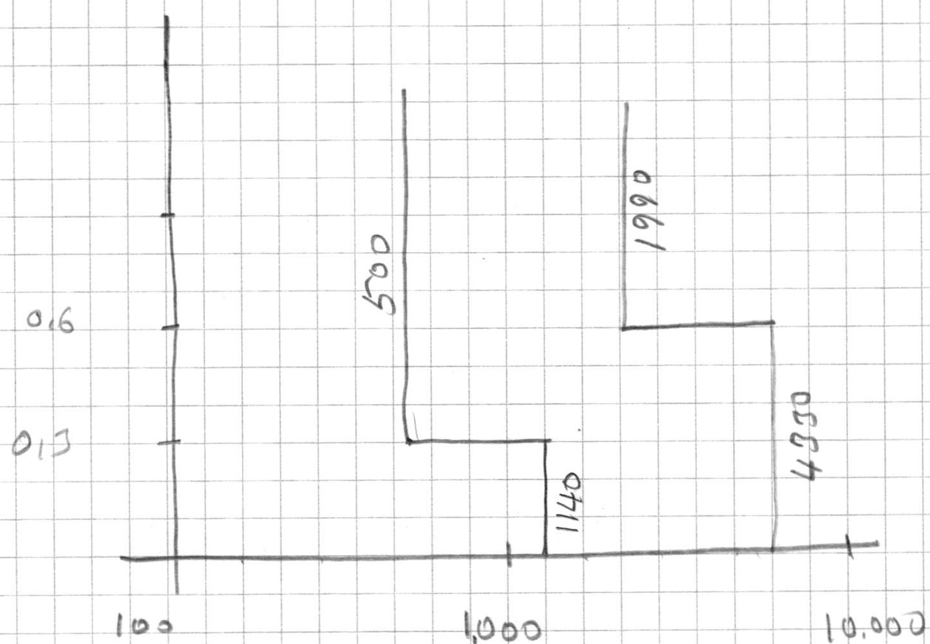
$$Z_{80\%\,L4} = \left[(Z_Q + Z_{T1} + Z_{L1}) // (Z_{T5} + Z_{G1}) + (Z_{T2} + Z_{L2})/2 \right] + (Z_{T4} + 0{,}8 \times Z_{L4})$$

$$Z_{80\%\,L4} = 1{,}1618 + j\,12{,}254$$

$$I_{>>6} = \frac{1.1 \times 22}{\sqrt{3} \cdot Z_{80\%\,L4}} = 1135 \approx 1140\text{ A}$$



Don't forget to refer the currents to their respective voltage levels.

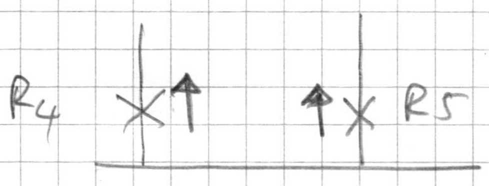
For R3, we have

$$I_{>3} = 1990 \times \frac{22}{132} = 330\text{ A} \qquad I_{>>3} = 720\text{ A}$$

$I>6 = 500 \times \frac{22}{6} = 1820A$ $\quad I>>6 = 4160\ A$

(c) Inrush currents are unsymmetrical, i.e. they have considerable 2. harmonics. R2 and R3 should suppress the 2. harmonic current.

(d) R4 and R5 should be directional, pointing upwards.



(e) We can install a differential relay at the terminals of each transformer. This way the diff relay can trip instantaneously in case there is a fault in the transformer.

Question 2 :

(a) Considering distance relays:

	R_{12}	R_{21}	R_{23}	R_{32}	R_{34}	R_{43}	R_{41}	R_{14}
Z_1	•							
Z_2	•	•					•	
Z_3	•	•		•	•		•	•

R_{12} will trip instantaneously and R_{21} will trip after Δt_2.

After R_{12} trips, R_{14}, R_{41} and R_{34} will reset.

After R_{21} trips, R_{32} will reset.

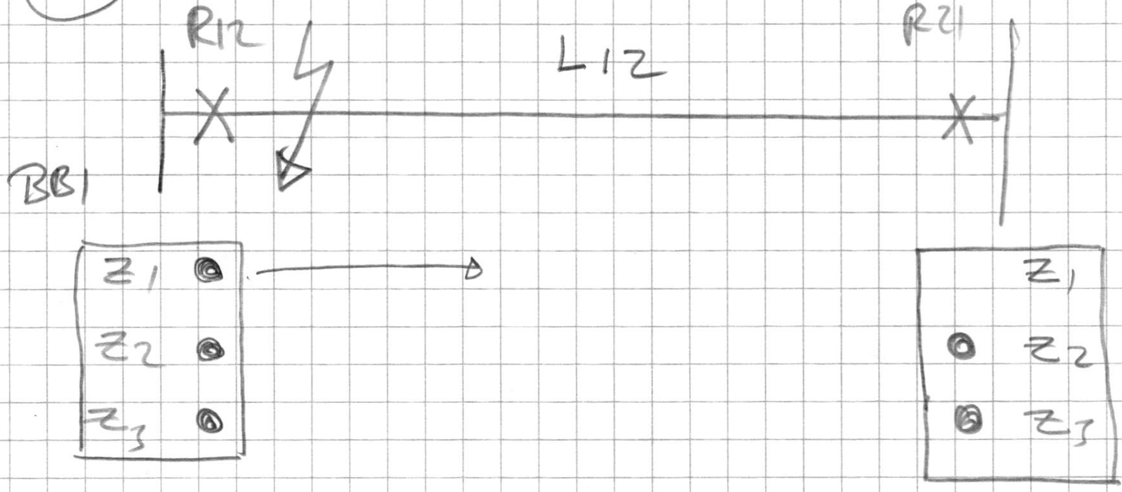
Regarding overcurrent relays, they must have a delay time greater than Δt_2 in order not to interfere with the distance relays so they will reset.

(b) In case R12 is out of order, R21 will trip after Δt_2. R41 sees the fault in its Z_2 so it will trip.

R14, R34 will reset after R41 and R32 will trip after R21.

Now G1 is still feeding the fault. Therefore R1 must trip after (say) $\Delta t_3 > \Delta t_2$.

(C)



PUTT = Permissive Underreach Transfer Trip.

Underreach means Z1.

R12 sends a Z1 signal to R21. Knowing that R21 has the fault in its Z2, it decides that the fault must be on L12 so it trips without waiting for Δt2.

If fault was on L41 near R14, R21 would still pick up the fault in its Z2 but in this case it would not have received Z1 signal from R12 so it would understand that the fault was NOT on L12.

Question 2 (Alt. 2)

a) Here we have fault at L41, near BB4.

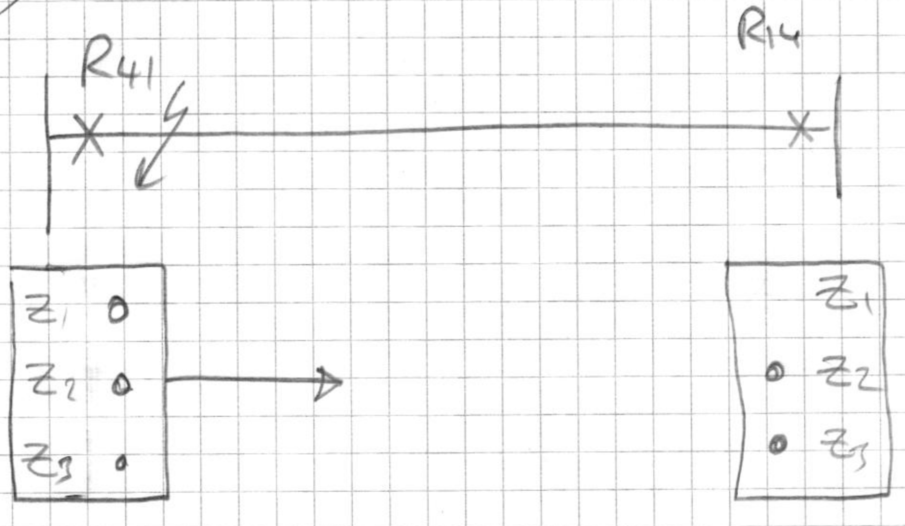
	R_{12}	R_{21}	R_{23}	R_{32}	R_{34}	R_{43}	R_{41}	R_{14}
Z_1							•	
Z_2					•		•	•
Z_3		•	•		•	•	•	•

R_{41} trips immediately, causing R_{43}, R_{34}, R_{23} to reset.

R_{14} trips after Δt and all the rest of the relays reset.

b) In case R_{41} does not work, R_{32} trips after Δt. R_{23} and R_{43} reset.

c) POTT: Permissive Overreach Transfer Trip



(overreach = Z2)

R_{41} sends a Z_2 signal to R_{14}. R_{14} has a Z_2 fault itself and decides to trip without further delay.

If the fault was behind R41, it would only pick it up in its Z_3 and would not send a Z_2 signal to R14. In this case R14 would wait.

Power Electronics, December 2020

Exercise 3.1. For a single-phase single-pulse thyristor rectifier with resistive load (R=15 Ohms), shown in Fig. 3.1, fed from an AC source with the voltage $v_s(t) = 220 \cdot \sin(314 \cdot t)$.

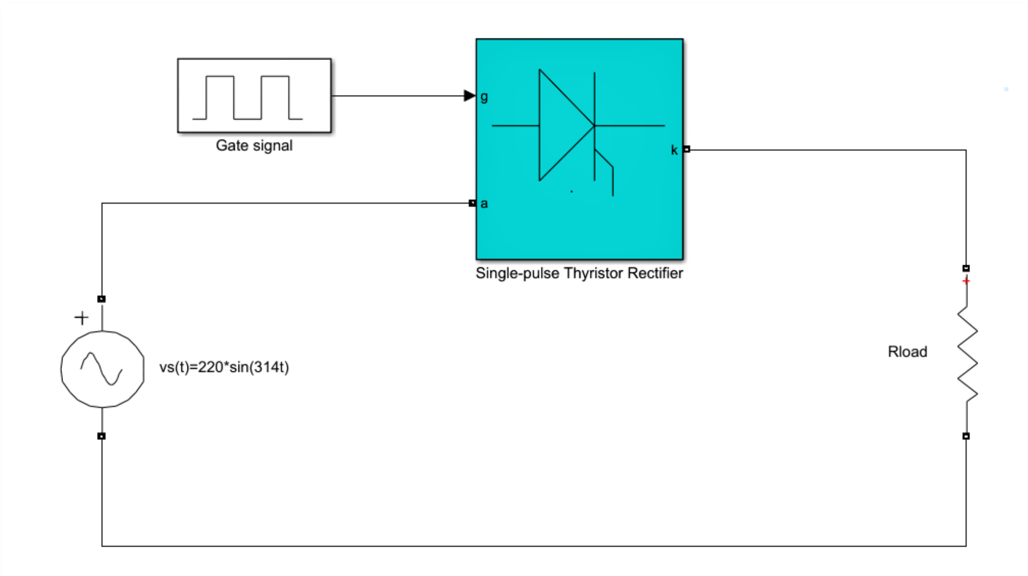


Fig. 3.1. Single-phase single-pulse thyristor rectifier

a) Explain when the average output voltage becomes maximum and when the RMS output voltage becomes zero.

The load average voltage:

$$V_{d\alpha} = \frac{1}{2\pi} \cdot \int_{\alpha}^{\pi} V_{\max} \cdot \sin(\omega t) \cdot d(\omega t) = \frac{V_{\max}}{2\pi} \cdot (1 + \cos(\alpha))$$

$$when\_\alpha = 0 \Rightarrow V_{d\alpha} = \frac{V_{\max}}{\pi}$$

The load/output average voltage can be varied from V_{max}/pi (when α=0) to zero (when α=π), by varying α from zero to π. The average output voltage becomes maximum when α=0.

$$V_{RMS} = \sqrt{\frac{1}{2\pi} \cdot \int_{\alpha}^{\pi} (V_{\max} \cdot \sin(\omega t))^2 \cdot d(\omega t)} = \frac{V_{\max}}{2} \sqrt{\frac{1}{\pi}\left[\pi - \alpha + \frac{\sin(2\alpha)}{2}\right]}$$

The load RMS voltage: $When\_\alpha = 0, V_{RMS} = \frac{V_{\max}}{2}$

$$When\_\alpha = \pi, V_{RMS} = 0$$

b) Assuming that the average output voltage is 70% of the maximum possible output voltage, calculate the firing angle (delay angle) and the efficiency of rectification ratio.

$$V_{d\alpha} = 0.7 \cdot V_{\max} = \frac{V_{\max}}{2\pi} \cdot (1+\cos(\alpha)) \Rightarrow \frac{1+\cos(\alpha)}{2} = 0.7$$

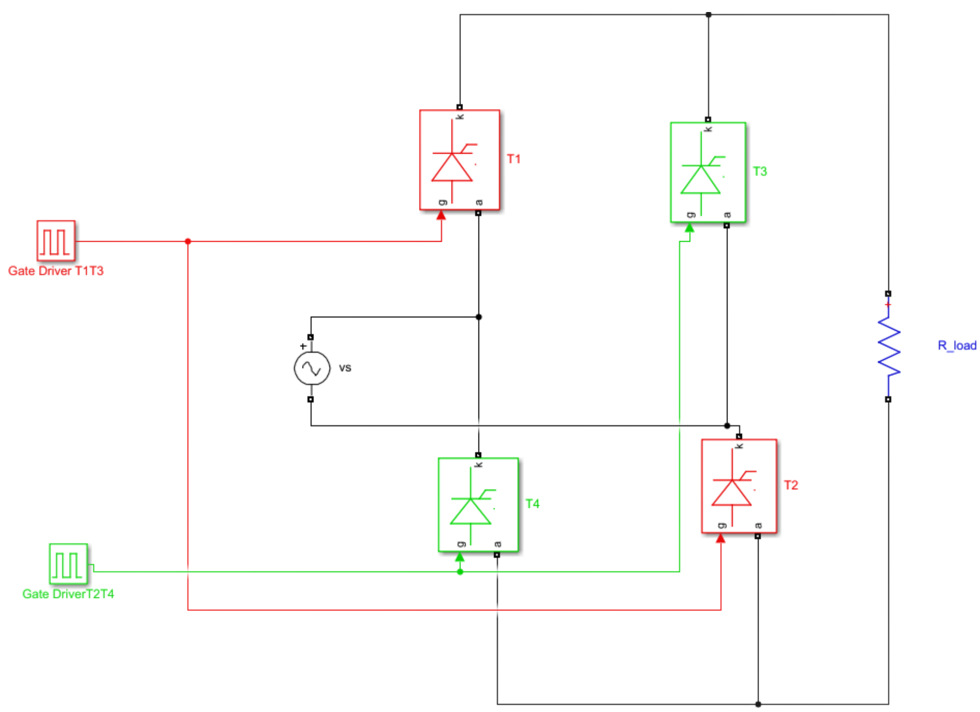
$$\cos(\alpha) = 0.4 \Rightarrow \alpha = 66.42^{\circ} \approx 1.16 rad$$

$$V_{d\alpha} = 0.7 \cdot V_{\max} = \frac{0.7 \cdot 220}{\pi} = 49.02[V]; I_d = \frac{V_d}{R} = \frac{49.02}{15} = 3.268[A]$$

$$V_{rms} = \frac{V_{\max}}{2}\sqrt{\frac{1}{\pi}\left[\pi - \alpha + \frac{\sin(2\alpha)}{2}\right]} = 95.12[V]; I_{rms} = \frac{V_{rms}}{R} = 6.34[A]$$

$$\eta = \frac{P_{dc}}{P_{ac}} = \frac{V_{d\alpha} \cdot I_d}{V_{rms} \cdot I_{rms}} = 26.56\%$$

c) Replace the single-pulse thyristor rectifier with a **full-bridge thyristor rectifier** and calculate the efficiency of rectification for this case/topology in the same conditions.



Single-phase full-bridge thyristor rectifier with resistive load.

$$For\_\alpha = 66.42^\circ : V_{d\alpha} = 0.7 \cdot V_{\max} = 0.7 \cdot \frac{2 \cdot 220}{\pi} = 98.04$$

$$I_d = \frac{98.04}{15} = 6.536$$

$$V_{rms} = \frac{220}{\sqrt{2}} \sqrt{\left[1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2 \cdot \pi}\right]} = 124.06[V]; I_{rms} = \frac{V_{rms}}{R} = 8.27[A]$$

$$\eta = \frac{P_{dc}}{P_{ac}} = \frac{V_{d\alpha} \cdot I_d}{V_{rms} \cdot I_{rms}} = 62.5\%$$

d) Replacing the resistive load with a DC motor represented by an equivalent circuit (Fig. 3.2) and considering the load current constant and ripple-free, draw the output voltage and the input current (Is), indicating in the sketch when the thyristors are in conduction.

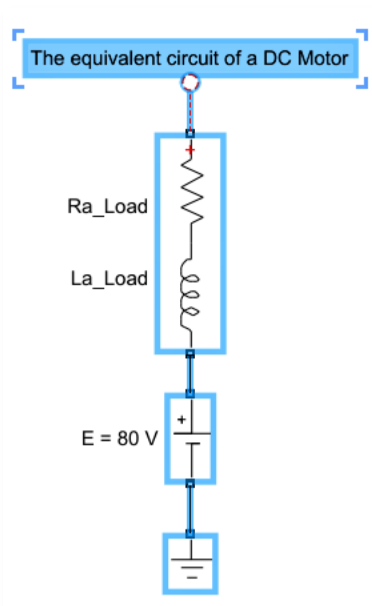
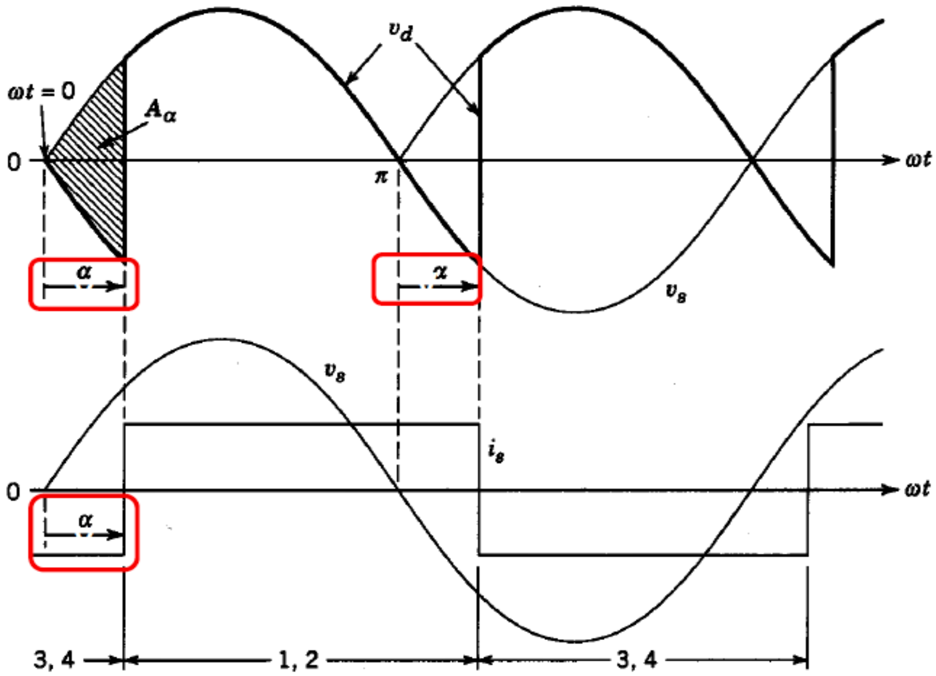


Fig. 3.2. The equivalent circuit of a DC motor.

Since the load is a DC motor, which is a highly inductive load, the load current is considered constant. The output voltage and the input current (Is) looks like in the next figure:



e) For the previous case, assuming that the speed is 1000 rpm, the armature resistance Ra=2 Ohms, the induced/back emf voltage of the motor E=80 V and the armature current is kept constant at Ia=10 A, find the firing angle for this case and for the case when the speed is 500 rpm.

Since the load is a DC motor, which is a highly inductive load, the load current is considered constant and equal with the average value of the load current (Id=Ia), as follows:

$$At\_1000rpm(E=80V), V_{d\alpha} = R_a \cdot I_a + E = \frac{2 \cdot 220}{\pi} \cdot \cos(\alpha) \Rightarrow \cos(\alpha) = \frac{2 \cdot 10 + 80}{140.06} = 0.714$$

$$\Rightarrow \alpha = 44.44^{\circ} = 0.77 rad$$

$$At\_500rpm(E=40V), V_{d\alpha} = R_a \cdot I_a + E/2 = \frac{2 \cdot 220}{\pi} \cdot \cos(\alpha) \Rightarrow \cos(\alpha) = \frac{2 \cdot 10 + 40}{140.06} = 0.428$$

$$\Rightarrow \alpha = 64.63^{\circ} = 1.13 rad$$

In a DC motor with constant current (Ia=ct), the speed is direct proportional with E.

$$\frac{E_1 \cdot I_a}{\omega_1} = \frac{E_2 \cdot I_a}{\omega_2}; \omega = \frac{2 \cdot \pi \cdot n}{60}$$

Exercise 4.1. A Hybrid Electric Vehicle (HEV) required a 20 kW half-bridge bidirectional converter (Fig. 4.1) to generate a 500 V from 200 V battery at a switching frequency fs=10 kHz.

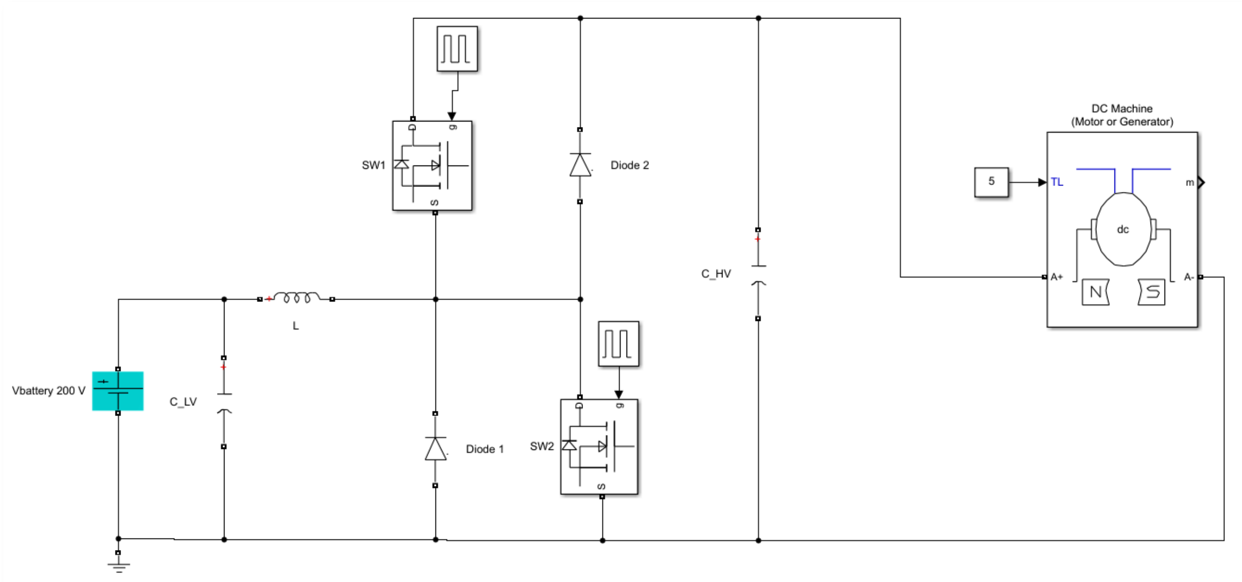


Fig. 4.1. DC machine drives by a half-bridge bidirectional converter.

a) Assuming that the electrical machine of HEV is working as generator to charge the low-voltage battery from the high-voltage DC-link, explain how the half-bridge converter works in this case and redraw the circuit such that to highlight the DC-DC converter type used for this case.

The buck and boost converters can be integrated together to create a bidirectional half-bridge converter. Since the machine is working in this case as generator or regenerative braking, the converter works as buck (step-down) converter to enable the low-voltage battery to be charged from a high-voltage DC-link/ input DC voltage source. The battery is seen by the converter as a load.

The converter from Fig. 4.1 can be redrawn as in Fig. 4.2 to create a buck converter, since the current is flowing in one direction from the generator (higher-voltage source) to the battery (lower voltage source). The active switch used in this case is SW1 (for ton state) and the passive switch is the Diode 1 (for toff state), from Fig. 4.1.

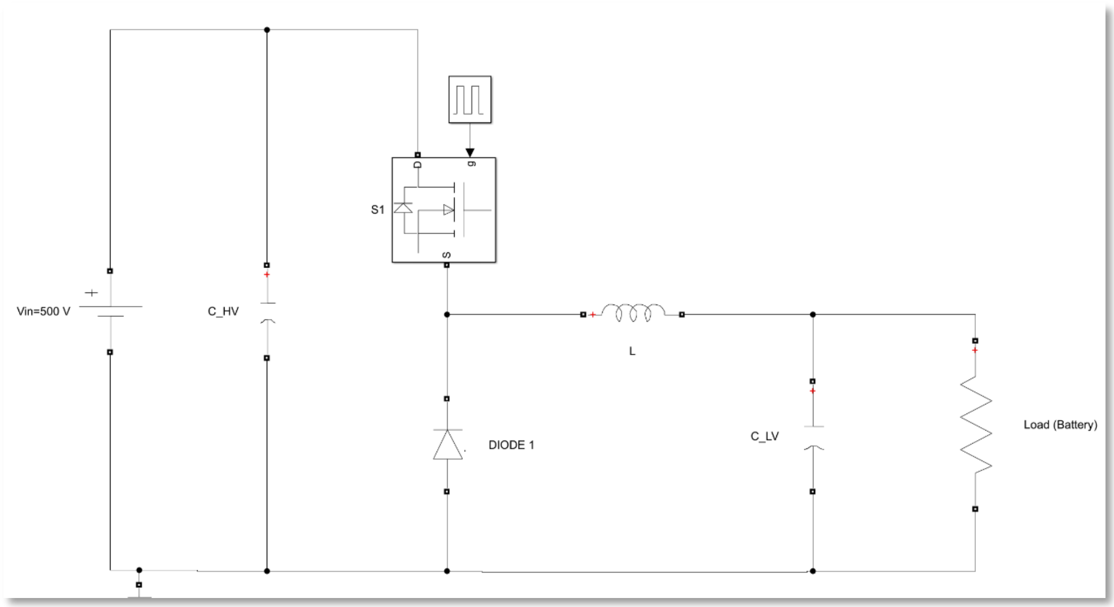


Fig. 4.2. Buck converter from the half-bridge converter.

b) For the DC-DC converter type, compatible with the generating operation mode, determine the components (L and C_{LV}) considering the inductor current ripple (28%) and the voltage ripple (0.5 %), assuming ideal components and ignoring the power loss. Assume that the converter is working in continuous conduction mode (CCM) and the converter sees the battery as a load (20 kW, 200 V).

For a buck converter: $D = \dfrac{V_o = V_{battery}}{V_{IN}} = \dfrac{200}{500} = 0.4$

The average inductor current: $I_L = I_o = \dfrac{P_o}{V_o} = \dfrac{20000}{200} = 100[A]$

The inductor current ripple (peak-peak): $\Delta i_L = 0.28 \cdot I_L = 0.28 \cdot 100 = 28[A]$

The desired inductance: $L = \dfrac{(V_{IN} - V_o) \cdot D}{f_s \cdot \Delta i_L} = \dfrac{(500 - 200) \cdot 0.4}{10000 \cdot 28} = 428.5[\mu H]$

The peak-to-peak value of the output voltage: $\Delta v_C = 0.005 \cdot 200 = 1[V]$

The low-voltage filter capacitance: $C_{LV} = \dfrac{\Delta i_L}{8 \cdot f_s \cdot \Delta v_C} = \dfrac{28}{8 \cdot 10000 \cdot 1} = 350[\mu F]$

c) Calculate the input current and the minimum and maximum inductor current for CCM

The input current: $I_{IN} = \frac{P_{IN}}{V_{IN}} = \frac{20000}{500} = 40[A]$

The max inductor current: $i_{L\max} = I_L + \frac{\Delta i_L}{2} = 100 + \frac{28}{2} = 114[A]$

The minimum inductor current: $i_{L\min} = I_L - \frac{\Delta i_L}{2} = 100 - \frac{28}{2} = 86[A]$

d) Determine the power level at which the converter enters BCM (at the boundary between CCM and DCM)

At the boundary, we can write:
$$I_{OB} = I_{LB} = \frac{\Delta i_L}{2} = 14[A]$$
$$P_B = V_O \cdot I_{OB} = 200 \cdot 14 = 2.8[kW]$$

$I_{LB(\max)} = 28[A], I_{LB(\min)} = 0$

e) As the load current and power is reduced, the converter works now in DCM (discontinuous conduction mode) for the given voltage conditions (Vemf=500 V, $V_{battery}$=200 V). Calculate the input and output currents and the inductor ripple current assuming the load power 2 kW for this case.

In DCM the load current is reduced to: $I_{O(DCM)} = \frac{P_O}{V_O} = \frac{2000}{200} = 10[A]$

The duty cycle becomes: $D = \sqrt{\frac{2 \cdot V_O}{V_{IN} \cdot (V_{IN} - V_O)} \cdot f_s \cdot L \cdot I_O} = 0.338$

The inductor current ripple: $\Delta i_L = \frac{(V_{IN} - V_O) \cdot D}{f_s \cdot L} = 23.66[A]$

The input current: $I_{IN} = \frac{P_{IN} = P_O}{V_{IN}} = \frac{2000}{500} = 4[A]$

Power Electronics-Second variant, December 2020

Exercise 3.2. For a single-phase single-pulse thyristor rectifier with resistive load (R=5 Ohms), shown in Fig. 3.1, fed from an AC source with the voltage $v_s(t) = 220 \cdot \sin(314 \cdot t)$.

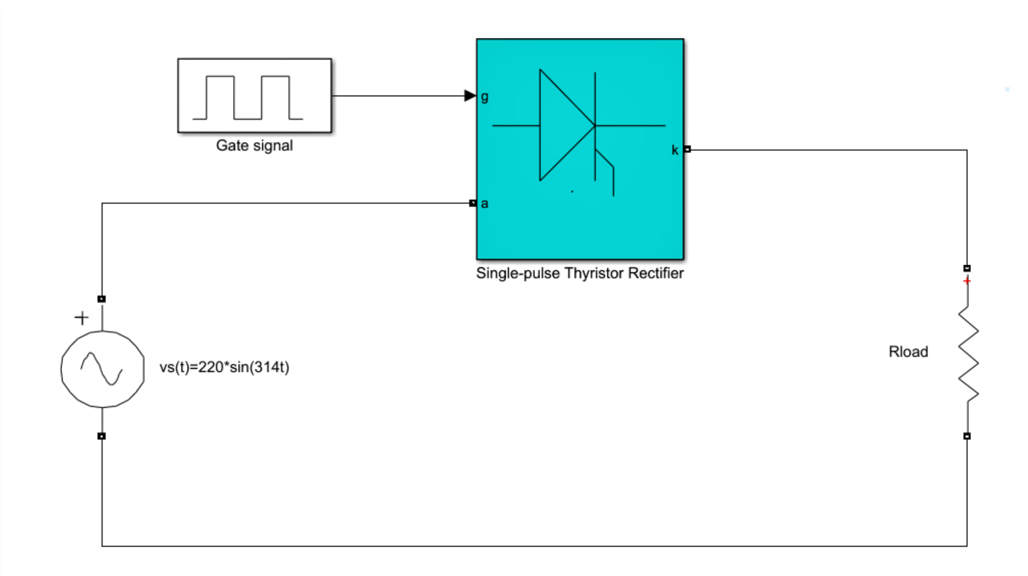


Fig. 3.1. Single-phase single-pulse thyristor rectifier

a) Explain when the average output voltage becomes maximum and when the RMS output voltage becomes zero.

The load/output average voltage:

$$V_{d\alpha} = \frac{1}{2\pi} \cdot \int_{\alpha}^{\pi} V_{\max} \cdot \sin(\omega t) \cdot d(\omega t) = \frac{V_{\max}}{2\pi} \cdot (1 + \cos(\alpha))$$

$$when\_\alpha = 0 \Rightarrow V_{d\alpha} = \frac{V_{\max}}{\pi}$$

The load average voltage can be varied from V_{max}/pi (when α=0) to zero (when α=π), by varying α from zero to π. The average output voltage becomes maximum when α=0.

$$V_{RMS} = \sqrt{\frac{1}{2\pi} \cdot \int_{\alpha}^{\pi} (V_{\max} \cdot \sin(\omega t))^2 \cdot d(\omega t)} = \frac{V_{\max}}{2} \sqrt{\frac{1}{\pi} \left[\pi - \alpha + \frac{\sin(2\alpha)}{2} \right]}$$

The load RMS voltage: $When\_\alpha = 0, V_{RMS} = \frac{V_{\max}}{2}$

$$When\_\alpha = \pi, V_{RMS} = 0$$

b) Assuming that the average output voltage is 80% of the maximum possible output voltage, calculate the firing angle (delay angle) and the efficiency of rectification ratio.

$$V_{d\alpha} = 0.7 \cdot V_{\max} = \frac{V_{\max}}{2\pi} \cdot (1+\cos(\alpha)) \Rightarrow \frac{1+\cos(\alpha)}{2} = 0.8$$

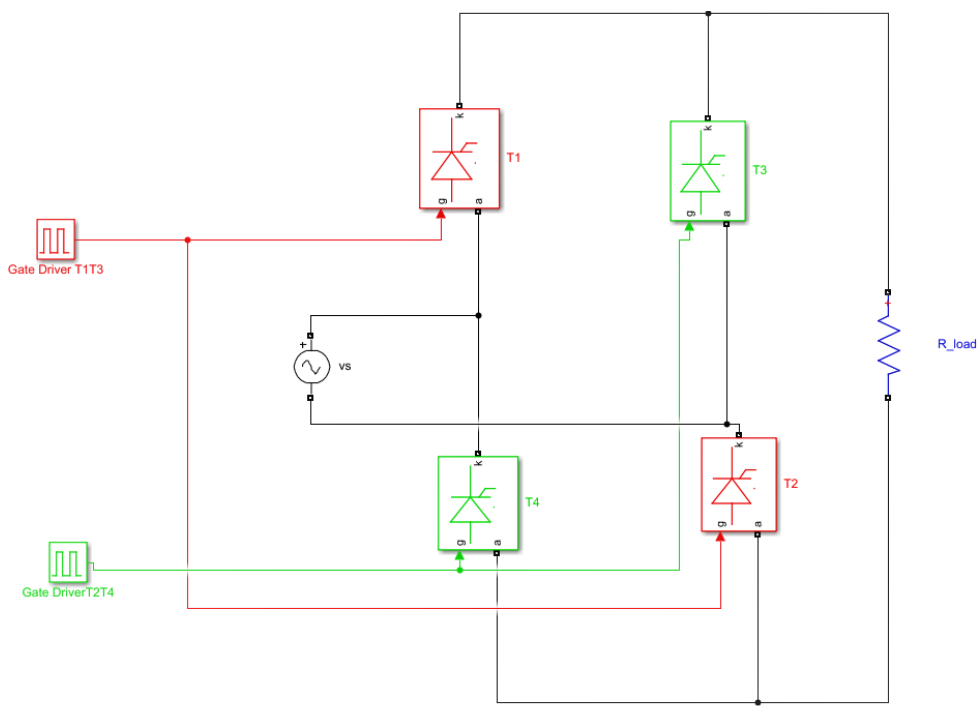
$$\cos(\alpha) = 0.6 \Rightarrow \alpha = 53.13^{\circ} \approx 0.93 rad$$

$$V_{d\alpha} = 0.8 \cdot V_{\max} = \frac{0.8 \cdot 220}{\pi} = 56.02[V]; I_d = \frac{V_d}{R} = \frac{56.02}{5} = 11.2[A]$$

$$V_{rms} = \frac{V_{\max}}{2}\sqrt{\frac{1}{\pi}\left[\pi - \alpha + \frac{\sin(2\alpha)}{2}\right]} = 92.36[V]; I_{rms} = \frac{V_{rms}}{R} = 18.47[A]$$

$$\eta = \frac{P_{dc}}{P_{ac}} = \frac{V_{d\alpha} \cdot I_d}{V_{rms} \cdot I_{rms}} = 37\%$$

c) Replace the single-pulse thyristor rectifier with a full-bridge thyristor rectifier and calculate the efficiency of rectification for this case/topology in the same conditions.



Single-phase full-bridge thyristor rectifier with resistive load.

$$For\_\alpha = 53.13^{\circ} : V_{d\alpha} = 0.8 \cdot V_{\max} = 0.8 \cdot \frac{2 \cdot 220}{\pi} = 112.045$$

$$I_d = \frac{112.045}{5} = 22.41$$

$$V_{rms} = \frac{220}{\sqrt{2}} \sqrt{\left[1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2 \cdot \pi}\right]} = 130.61[V]; I_{rms} = \frac{V_{rms}}{R} = 26.12[A]$$

$$\eta = \frac{P_{dc}}{P_{ac}} = \frac{V_{d\alpha} \cdot I_d}{V_{rms} \cdot I_{rms}} = \frac{2511}{3412} = 74\%$$

d) Replacing the resistive load with a DC motor represented by an equivalent circuit (Fig. 3.2) and considering the load current constant and ripple-free, draw the output voltage and the input current (Is), indicating in the sketch when the thyristors are in conduction.

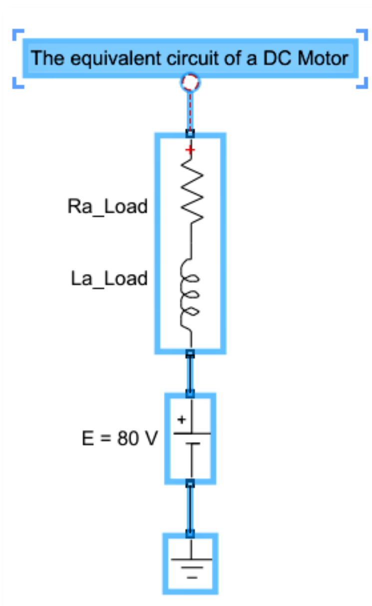
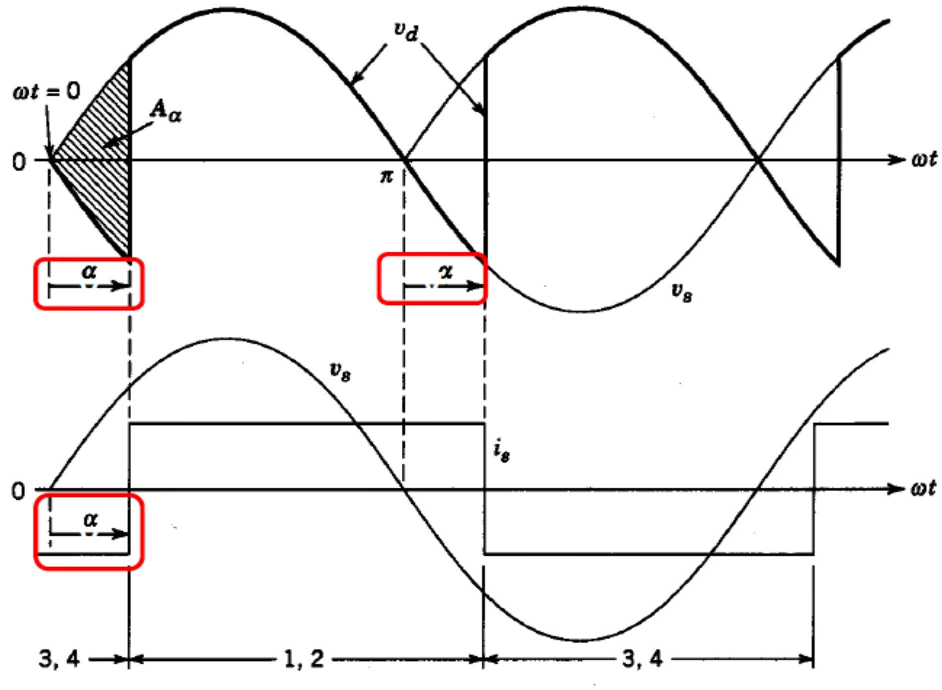


Fig. 3.2. The equivalent circuit of a DC motor.

Since the load is a DC motor, which is a highly inductive load, the load current is considered constant. The output voltage and the input current (Is) looks like in the next figure:



e) For the previous case, assuming that the speed is 1000 rpm, the armature resistance Ra=2 Ohms, the induced/back emf voltage of the motor E=80 V and the armature current is kept constant at Ia=10 A, find the firing angle for this case and for the case when the speed is 500 rpm.

Since the load is a DC motor, which is a highly inductive load, the load current is considered constant and equal with the average value of the load current (Id=Ia), as follows:

$$At\_1000rpm(E=80V), V_{d\alpha} = R_a \cdot I_a + E = \frac{2\cdot 220}{\pi}\cdot\cos(\alpha) \Rightarrow \cos(\alpha) = \frac{2\cdot 10+80}{140.06} = 0.714$$

$$\Rightarrow \alpha = 44.44^\circ = 0.77rad$$

$$At\_500rpm(E=40V), V_{d\alpha} = R_a \cdot I_a + E/2 = \frac{2\cdot 220}{\pi}\cdot\cos(\alpha) \Rightarrow \cos(\alpha) = \frac{2\cdot 10+40}{140.06} = 0.428$$

$$\Rightarrow \alpha = 64.63^\circ = 1.13rad$$

In a DC motor with constant current (Ia=ct), the speed is direct proportional with E.

$$\frac{E_1\cdot I_a}{\omega_1} = \frac{E_2\cdot I_a}{\omega_2}; \omega = \frac{2\cdot\pi\cdot n}{60}$$

Exercise 4.2. A Hybrid Electric Vehicle (HEV) required a 20 kW half-bridge bidirectional converter (Fig. 4.1) to generate a 500 V from 200 V battery at a switching frequency fs=10 kHz.

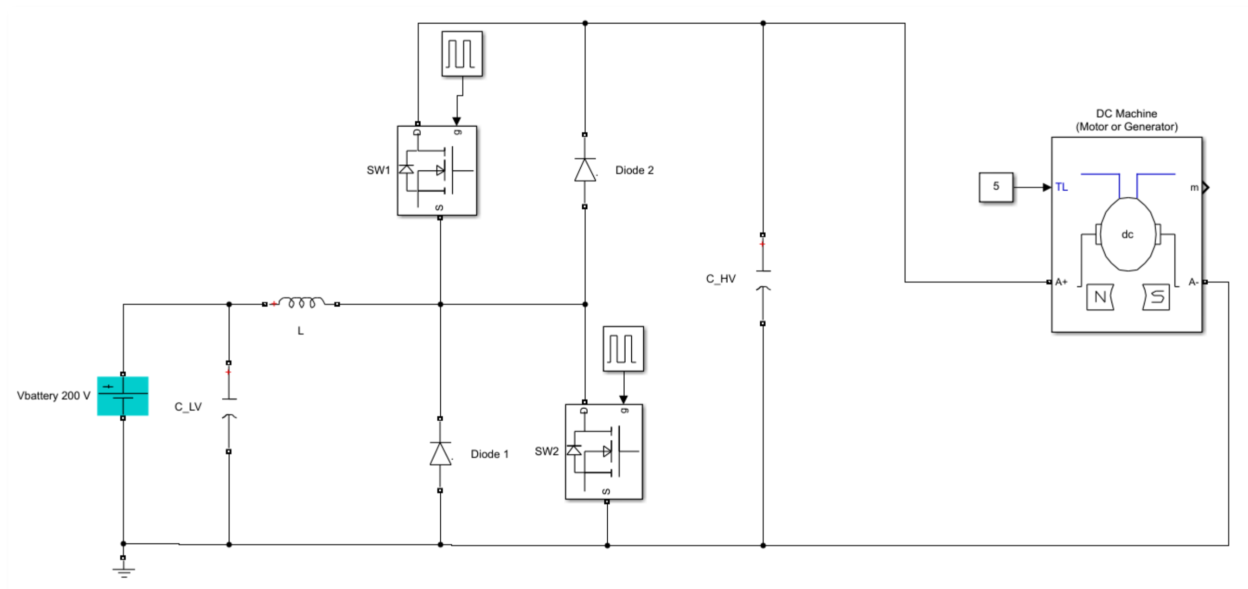


Fig. 4.1. DC machine drives by a half-bridge bidirectional converter.

a) Assuming that the DC machine of HEV is working as a motor enabling to discharge the low-voltage battery to a higher voltage DC-link, explain how the half-bridge converter works in this case and redraw the circuit such that to highlight the DC-DC converter type used for this case.

Solution: The buck and boost converters can be integrated together to create a bidirectional half-bridge converter. Since the machine is working as a motor in this case, the converter works as boost (step-up) converter to enable the low-voltage battery to be discharged over a higher DC voltage-link (supplying the motor from a lower voltage source). The motor is seen by the converter as a load.

The converter from Fig. 4.1 can be redrawn as in Fig. 4.2 to create a boost converter, as the current is flowing in one direction, from the lower voltage source to a higher DC-link voltage. The active switch used in this case is SW2 (during ton state) and the passive switch is the Diode 2 (for toff state), from Fig. 4.1.

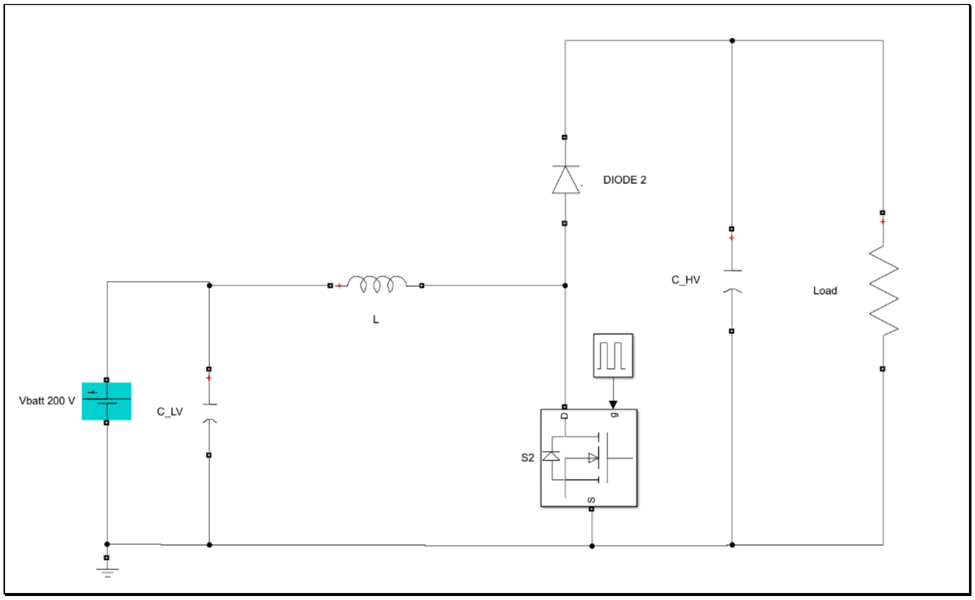


Fig. 4.2. Boost converter as part of the half-bridge converter.

b) For the DC-DC converter type, compatible with the motoring operation mode, determine the components (L and C_{HV}) considering the inductor current ripple (28%) and the voltage ripple (0.5 %), assuming ideal components and ignoring the power loss. Assume that the converter is working in continuous conduction mode (CCM) and the converter sees the motor as a resistive load (20 kW, 500 V).

Solution:

For a boost converter the duty ratio: $D = 1 - \frac{V_{IN} = V_{battery}}{V_O} = 1 - \frac{200}{500} = 1 - 0.4 = 0.6$

The average inductor current: $I_L = I_{IN} = \frac{P_{IN}}{V_{IN}} = \frac{20000}{200} = 100[A]$

The inductor current ripple (peak-peak): $\Delta i_L = 0.28 \cdot I_L = 0.28 \cdot 100 = 28[A] = \frac{V_{IN} \cdot D}{f_s \cdot L}$

The desired inductance: $L = \frac{V_{IN} \cdot D}{f_s \cdot \Delta i_L} = \frac{200 \cdot 0.6}{10000 \cdot 28} = 428.5[\mu H]$

The peak-to-peak value of the output voltage: $\Delta v_C = 0.005 \cdot 500 = 2.5[V]$

The high-voltage filter capacitance:

$$C_{HV} = \frac{I_O \cdot D}{f_s \cdot \Delta v_C} = \frac{40 \cdot 0.6}{10000 \cdot 2.5} = 960[\mu F]$$

$$where\_I_O = \frac{P_O}{V_O} = \frac{20000}{500} = 40[A]$$

c) Calculate the input current and the minimum and maximum inductor current for CCM

Solution: The input current was calculated before

The max inductor current: $i_{L\max} = I_L + \frac{\Delta i_L}{2} = 100 + \frac{28}{2} = 114[A]$

The minimum inductor current: $i_{L\min} = I_L - \frac{\Delta i_L}{2} = 100 - \frac{28}{2} = 86[A]$

d) Determine the power level at which the converter enters BCM (at the boundary between CCM and DCM)

Solution:

At the boundary, we can write:

$$I_{INB} = I_{LB} = \frac{\Delta i_L}{2} = 14[A]$$

$$P_B = V_{IN} \cdot I_{INB} = 200 \cdot 14 = 2.8[kW]$$

e) As the load current and power is reduced, the converter works now in DCM (discontinuous conduction mode) for the given voltage conditions (Vemf=500 V). Calculate the input and output currents and the inductor ripple current assuming the load power 2 kW for this case.

Solution: In DCM the load current is reduced to: $I_{O(DCM)} = \frac{P_O}{V_O} = \frac{2000}{500} = 4[A]$

The duty cycle becomes: $D = \sqrt{\frac{(V_O - V_{IN})}{V_{IN} \cdot V_O} \cdot 2 \cdot f_s \cdot L \cdot I_{IN}} = 0.507$

The inductor current ripple: $\Delta i_L = \frac{V_{IN} \cdot D}{f_s \cdot L} = 23.66[A]$

The input current: $I_{IN} = \frac{P_{IN} = P_O}{V_{IN}} = \frac{2000}{200} = 10[A]$