Høgskolen i Østfold

EKSAMEN

Emnekode:	Emnenavn:
IRM20015	Mekanikk 2 Deleksamen 2: Fasthetslære
Dato: 16.12.2019 **Sensurfrist:** 27.12.2019	**Eksamenstid:** KL 0900 - 1200
Antall oppgavesider: 3 **Antall vedleggsider:** 4	**Faglærer:** Jeovan De Freitas (970 968 317) **Oppgaven er kontrollert:** Ja

Hjelpemidler:

- Kalkulator

Om eksamensoppgaven:

Alle besvarelser må begrunnes

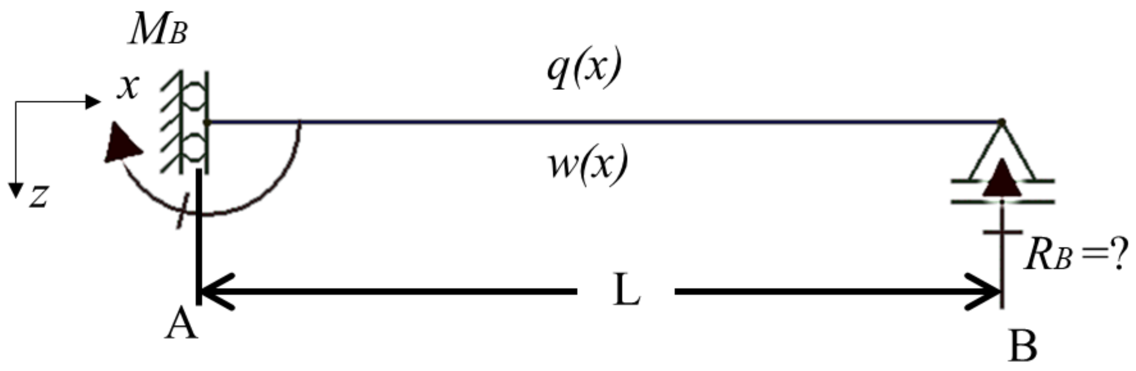
Kandidaten må selv kontrollere at oppgavesettet er fullstendig

Eksemen

Oppgave 1 (30%)

Hvis Bøyemomentskurve for den vist bjelke med tverrsnitts konstante *E* og *I* er:

$$M(x) \rightarrow \frac{p_0 \cdot (L^3 - x^3)}{6 \cdot L}$$



a) Bestem **Reaksjonskraft** R_B ved punkt B
b) Bjelkens **Lastkurve** ***q(x).***
c) Bjelkens **Utbøyningskurven** ***w(x)***
d) **Vinkelendringen** θ_B ved punkt B

a) Bestem **Reaksjonskraft** $\boldsymbol{R_B}$ ved punkt B

$$M(x) \rightarrow \frac{p_0 \cdot (L^3 - x^3)}{6 \cdot L}$$

$$V(x) := \frac{d}{dx} M(x) \xrightarrow{explicit, ALL} \frac{d}{dx} \frac{p_0 \cdot (L^3 - x^3)}{6 \cdot L} \rightarrow -\frac{p_0 \cdot x^2}{2 \cdot L}$$

$$x := 0 \qquad R_A := V(x) \xrightarrow{explicit, ALL} -\frac{p_0 \cdot 0^2}{2 \cdot L} \rightarrow 0$$

$$x := L \qquad R_B := V(x) \xrightarrow{explicit, ALL} -\frac{p_0 \cdot L^2}{2 \cdot L} \rightarrow -\frac{p_0 \cdot L}{2}$$

b) Bjelkens **Lastkurve** ***q(x).***

$$V(x) \rightarrow -\frac{p_0 \cdot x^2}{2 \cdot L}$$

$$q(x) := \frac{d}{dx} V(x) \xrightarrow{explicit, ALL} \frac{d}{dx} -\frac{p_0 \cdot x^2}{2 \cdot L} \rightarrow -\frac{p_0 \cdot x}{L}$$

c) Bjelkens **Utbøyningskurven** ***w(x)***

$$M(x) \rightarrow \frac{p_0 \cdot (L^3 - x^3)}{6 \cdot L}$$

$$EI \cdot \frac{d^2}{dx^2} w(x) = M(x) \rightarrow EI \cdot \frac{d^2}{dx^2} w(x) = \frac{p_0 \cdot (L^3 - x^3)}{6 \cdot L}$$

$$\frac{d}{dx} w(x) = \theta(x)$$

$$\theta(x) = \int \frac{M(x)}{EI} dx + C_1 \xrightarrow{explicit, ALL} \theta(x) = \int \frac{\frac{p_0 \cdot (L^3 - x^3)}{6 \cdot L}}{EI} dx + C_1 \rightarrow \theta(x) = C_1 + \frac{p_0 \cdot L^2 \cdot x}{6 \cdot EI} - \frac{p_0 \cdot x^4}{24 \cdot L \cdot EI}$$

$$C_1 := \theta(0) = 0 \rightarrow C_1 = 0 \xrightarrow{solve, C_1} 0$$

$$\theta(x) \rightarrow \frac{p_0 \cdot L^2 \cdot x}{6 \cdot EI} - \frac{p_0 \cdot x^4}{24 \cdot L \cdot EI}$$

$$w(x) = \int \theta(x) dx + C_2 \rightarrow w(x) = C_2 + \frac{p_0 \cdot L^2 \cdot x^2}{12 \cdot EI} - \frac{p_0 \cdot x^5}{120 \cdot L \cdot EI}$$

$$C_2 := w(L) = 0 \xrightarrow{explicit, ALL} C_2 + \frac{p_0 \cdot L^2 \cdot L^2}{12 \cdot EI} - \frac{p_0 \cdot L^5}{120 \cdot L \cdot EI} = 0 \rightarrow C_2 + \frac{3 \cdot p_0 \cdot L^4}{40 \cdot EI} = 0 \xrightarrow{solve, C_2} -\frac{3 \cdot p_0 \cdot L^4}{40 \cdot EI}$$

$$w(x) \rightarrow \frac{p_0 \cdot L^2 \cdot x^2}{12 \cdot EI} - \frac{3 \cdot p_0 \cdot L^4}{40 \cdot EI} - \frac{p_0 \cdot x^5}{120 \cdot L \cdot EI}$$

d) **Vinkelendringen θ_B** ved punkt B

$$M(x) \rightarrow \frac{p_0 \cdot (L^3 - x^3)}{6 \cdot L}$$

$$EI \cdot \frac{d^2}{dx^2} w(x) = M(x) \rightarrow EI \cdot \frac{d^2}{dx^2} w(x) = \frac{p_0 \cdot (L^3 - x^3)}{6 \cdot L}$$

$$\frac{d}{dx} w(x) = \theta(x)$$

$$\theta(x) = \int \frac{M(x)}{EI} dx + C_1 \xrightarrow{explicit, ALL} \theta(x) = \int \frac{\frac{p_0 \cdot (L^3 - x^3)}{6 \cdot L}}{EI} dx + C_1 \rightarrow \theta(x) = C_1 + \frac{p_0 \cdot L^2 \cdot x}{6 \cdot EI} - \frac{p_0 \cdot x^4}{24 \cdot L \cdot EI}$$

$$C_1 := \theta(0) = 0 \rightarrow C_1 = 0 \xrightarrow{solve, C_1} 0$$

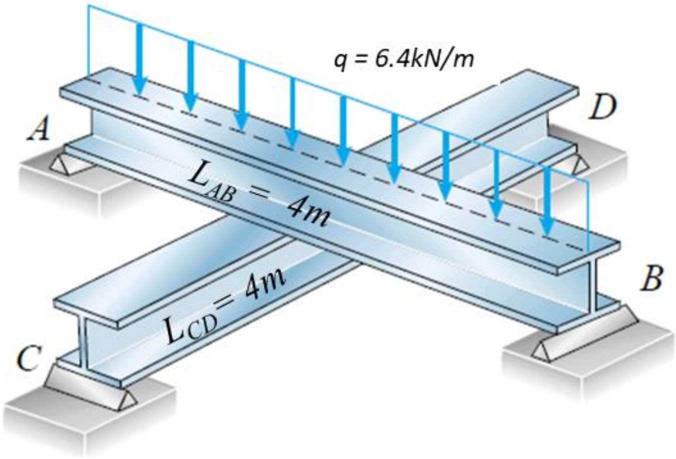
$$\theta(x) \rightarrow \frac{p_0 \cdot L^2 \cdot x}{6 \cdot EI} - \frac{p_0 \cdot x^4}{24 \cdot L \cdot EI}$$

$$x := L$$

$$\theta_B := \theta(L) \xrightarrow{explicit, ALL} \frac{p_0 \cdot L^2 \cdot L}{6 \cdot EI} - \frac{p_0 \cdot L^4}{24 \cdot L \cdot EI} \rightarrow \frac{p_0 \cdot L^3}{8 \cdot EI}$$

Oppgave 2 (30%)

To identiske fritt opplagte bjelker ***A-B*** og ***C-D*** er plassert slik at de krysser hverandre ved midtpunktene. I ubelastet tilstand er det akkurat kontakt mellom bjelkene i midtpunktet. Bjelke A-A belastes med en jevnt fordelt vertikal kraft ***q=6.4kN/m***, som viser på figur.



Vi antar at begge bjelkene har samme lengde, $L_{AB} = L_{CD} = L = 4m$, og samme Stivheten ***EI***.

Hvis vi antar at alle horisontale krefter er null, bestem:

a) **Reaksjonskrefter i Bjelke C-D;**

b) **Maksimalt bøyemoment i Bjelke A-B**.

c) **Maksimalt bøyemoment i Bjelke C-D**

Maks forskyvning i midtpunkt AB på grunn av last q i bjelke A-B (bjelkeformler 16)

$$\delta_{Aq} := \frac{5 \cdot q \cdot (L_{AB})^4}{384 \cdot E \cdot I} \rightarrow \frac{5 \cdot L_{AB}^{\ 4} \cdot q}{384 \cdot E \cdot I}$$

Maks forskyvning i midtpunkt AB på grunn av opplegg reaksjon X i bjelke A-B (bjelkeformler 11)

$$\delta_{AX} := \frac{X \cdot (L_{AB})^3}{48 \cdot E \cdot I} \rightarrow \frac{L_{AB}^{\ 3} \cdot X}{48 \cdot E \cdot I}$$

Total forskyvning i midtpunkt AB er $\Delta_{AB} := \delta_{Aq} - \delta_{AX} \rightarrow \frac{5 \cdot L_{AB}^{\ 4} \cdot q}{384 \cdot E \cdot I} - \frac{L_{AB}^{\ 3} \cdot X}{48 \cdot E \cdot I}$

Opplegg reaksjon X i bjelke A-B virke som punkt last i bjelke C-D, defor forskyvning i midtpunkt CD at bjelke C-D er: (bjelkeformler 11)

$L_{AB} = L_{CD}$

$$\Delta_{CD} := \frac{X \cdot (L_{CD})^3}{48 \cdot E \cdot I} \rightarrow \frac{L_{AB}^{\ 3} \cdot X}{48 \cdot E \cdot I}$$

Kinematiske betingelse er at forskyvning i punkt C må være den samme i begge bjelker

$\Delta_{AB} = \Delta_{CD}$

$$\Delta_{AB} = \Delta_{CD} \rightarrow \frac{5 \cdot L_{AB}^{\ 4} \cdot q}{384 \cdot E \cdot I} - \frac{L_{AB}^{\ 3} \cdot X}{48 \cdot E \cdot I} = \frac{L_{AB}^{\ 3} \cdot X}{48 \cdot E \cdot I} \xrightarrow{solve, X} \frac{5 \cdot L_{AB} \cdot q}{16}$$

$$X \rightarrow \frac{5 \cdot L_{AB} \cdot q}{16}$$

Med values:

$L_{AB} := 4\ m$ $\quad L_{BC} := 1.5\ m$ $\quad I_{AA} := \frac{48\ mm \cdot (198\ mm)^3}{12}$ $\quad q := 6.4\ \frac{kN}{m}$

$$X := \frac{5 \cdot L_{AB} \cdot q}{16} \xrightarrow{explicit, ALL} \frac{5 \cdot 4\ m \cdot 6.4\ \frac{kN}{m}}{16} = 8\ kN$$

$F_{midtCD} := X = 8\ kN$ $\quad R_{midtAB} = X \rightarrow R_{midtAB} = 8 \cdot kN$

a) **Reaksjonskrefter i Bjelke C-D;**

$$\Sigma M_C = 0$$

$$R_D := R_D \cdot L_{AB} - F_{midtCD} \cdot \frac{L_{AB}}{2} = 0 \xrightarrow{explicit, ALL} R_D \cdot 4\ m - 8 \cdot kN \cdot \frac{4\ m}{2} = 0 \xrightarrow{solve, R_D} 4 \cdot kN = 4\ kN$$

$$\Sigma F_y = 0$$

$$R_C := -R_D + F_{midtCD} \xrightarrow{explicit, ALL} -(4 \cdot kN) + 8 \cdot kN = 4\ kN$$

$R_D = 4\ kN$ $R_C = 4\ kN$

b) **Maksimalt bøyemoment i Bjelke A-B**.

Reaksjon i bjelke A-B

$$\Sigma M_A = 0$$

$$R_B := R_B \cdot L_{AB} + R_{midtAB} \cdot \frac{L_{AB}}{2} - q \cdot (L_{AB}) \cdot \left(\frac{L_{AB}}{2}\right) = 0 \xrightarrow{explicit, ALL} R_B \cdot 4\ m + 8 \cdot kN \cdot \frac{4\ m}{2} - 6.4\ \frac{kN}{m} \cdot 4\ m \cdot \frac{4\ m}{2} = 0 \xrightarrow{solve, R_B} 12.8 \cdot kN - 4.0 \cdot kN = 8.8\ kN$$

$$\Sigma F_y = 0$$

$$R_A := -R_B - R_{midtAB} + q \cdot L_{AB} \rightarrow 25.6 \cdot kN - 8 \cdot kN - 8.8 \cdot kN = 8.8\ kN$$

$$R_A = 8.8\ kN \qquad R_B = 8.8\ kN$$

Momentkurve x<L/2

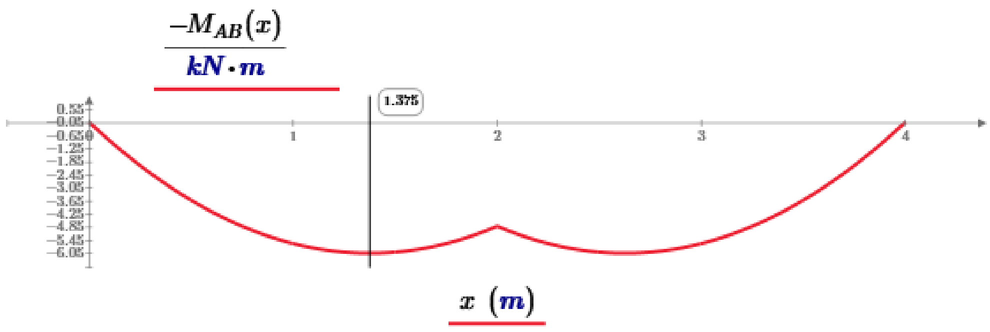
$$M(x) := R_A \cdot x - q \cdot x \cdot \frac{x}{2} \qquad V(x) := \frac{d}{dx} M(x) \rightarrow 8.8 \cdot kN - \frac{6.4 \cdot x \cdot kN}{m}$$

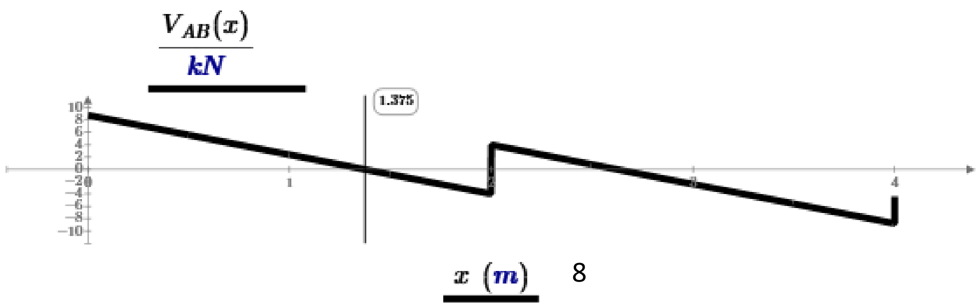
Moment maks V=0

$$x_{maks} := V(x) = 0 \rightarrow 8.8 \cdot kN - \frac{6.4 \cdot x \cdot kN}{m} = 0 \xrightarrow{solve, x} \frac{1.375 \cdot m \cdot kN}{kN} \qquad x_{maks} = 1.375\ m$$

$$M_{maks} := M(x_{maks}) \xrightarrow{explicit, ALL} 8.8 \cdot kN \cdot \frac{1.375 \cdot m \cdot kN}{kN} - 6.4\ \frac{kN}{m} \cdot \frac{1.375 \cdot m \cdot kN}{kN} \cdot \frac{\frac{1.375 \cdot m \cdot kN}{kN}}{2} = 6.05\ kN \cdot m$$

$$M_{maks} = 6.05\ kN \cdot m$$





c) **Maksimalt bøyemoment i Bjelke C-D**

Momentkurve x<L/2

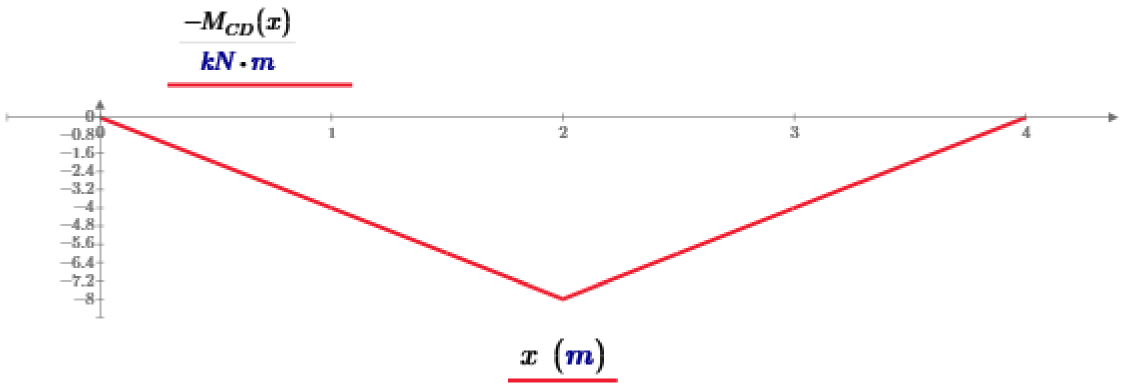
$M(x) := R_D \cdot x \rightarrow 4 \cdot x \cdot kN$ $\qquad$ $V(x) := \frac{d}{dx} M(x) \rightarrow 4 \cdot kN$

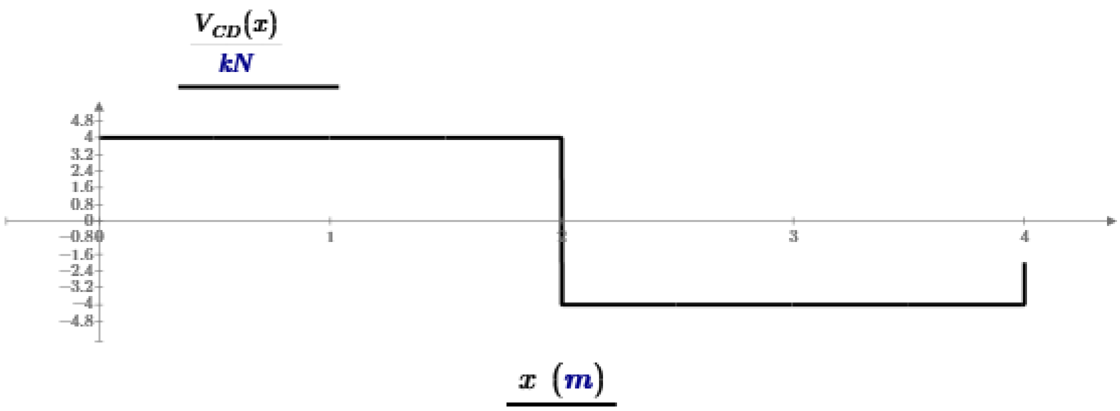
Moment maks xL/2

$$M_{maksCD} := M\left(\frac{L_{CD}}{2}\right) \xrightarrow{explicit, ALL} 4 \cdot \frac{4\ m}{2} \cdot kN = 8\ kN \cdot m$$

$M_{maksCD} = 8\ kN \cdot m$ eller:

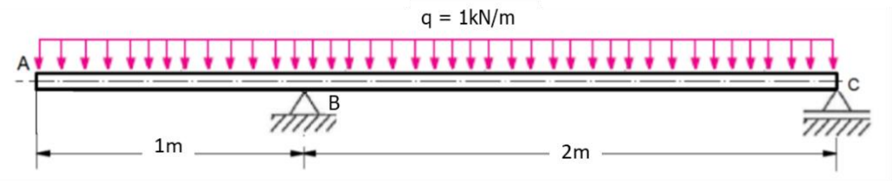
$$M_{maksCD} := F_{midtCD} \cdot \frac{L_{CD}}{4} \xrightarrow{explicit, ALL} 8 \cdot kN \cdot \frac{4\ m}{4} = 8\ kN \cdot m$$

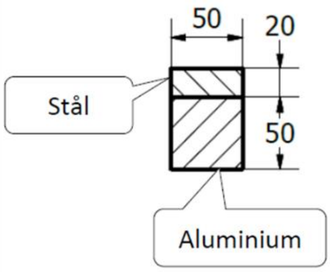




Oppgave 3 (40%)

En komposittbjelke AC har et profil som vist på figuren til Høyre. Den er satt sammen av et profil laget i aluminium, med stålforsterkning over. E-modulen for aluminium er $\boldsymbol{E_{alu}=70\,000\ MPa}$ og E-modulen for stål er $\boldsymbol{E_{stål}=210\,000\ MPa}$.





a) Hva er **annet arealmoment** når man bruker aluminium som basismatriale?

b) Hva er **størst spenning $\sigma_{stål}$ i stålet**?

c) Hva er **størst spenning σ_{alu} i aluminiumen**?

d) Hva er **maksimum skjærspenning** τ_{maks} mellom stål og aluminiumen?

a) Hva er **annet arealmoment** når man bruker aluminium som basismatriale?

$$E_{stål} := 210000\ MPa \quad E_{alu} := 70\ GPa = 70000\ MPa$$

$$b_{alu} := 50 \cdot mm \quad h_{alu} := 50 \cdot mm \quad b_{stål} := 50\ mm \quad h_{stål} := 20\ mm$$

$$n := \frac{E_{stål}}{E_{alu}} = 3 \qquad b_{stål\_alu} := b_{stål} \cdot n \xrightarrow{explicit,ALL} 50\ mm \cdot 3 = 150\ mm$$

$$A_{stål} := b_{stål\_alu} \cdot h_{stål} = \left(3 \cdot 10^{3}\right)\ mm^{2} \qquad I_{stål} := \frac{b_{stål\_alu} \cdot h_{stål}^{\ 3}}{12} = \left(1 \cdot 10^{5}\right)\ mm^{4}$$

$$A_{alu} := b_{alu} \cdot h_{alu} \rightarrow 2500 \cdot mm^{2} \qquad I_{alu} := \frac{b_{alu} \cdot h_{alu}^{\ 3}}{12} = \left(5.21 \cdot 10^{5}\right)\ mm^{4}$$

$$z_1 := \frac{h_{stål}}{2} + h_{alu} = 60\ mm \qquad z_2 := \frac{h_{alu}}{2} = 25\ mm$$

$$z := \frac{A_{stål} \cdot z_1 + A_{alu} \cdot z_2}{A_{stål} + A_{alu}} \xrightarrow{explicit,ALL} \frac{3000 \cdot mm^{2} \cdot 60 \cdot mm + 2500 \cdot mm^{2} \cdot 25 \cdot mm}{3000 \cdot mm^{2} + 2500 \cdot mm^{2}} = 44.1\ mm$$

$$z = 44\ mm$$

$$y_1 := z_1 - z = 16\ mm \qquad y_2 := z - z_2 = 19\ mm$$

$$I_t := I_{stål} + A_{stål} \cdot y_1^{\ 2} + I_{alu} + A_{alu} \cdot y_2^{\ 2} \xrightarrow{explicit,ALL} 100000 \cdot mm^{4} + 3000 \cdot mm^{2} \cdot (16 \cdot mm)^{2} + 520833 \cdot mm^{4} + 2500 \cdot mm^{2} \cdot (19 \cdot mm)^{2} = \left(2.29 \cdot 10^{6}\right)\ mm^{4}$$

$$I_t = \left(2.291 \cdot 10^{6}\right)\ mm^{4}$$

b) Hva er **størst spenning $\sigma_{stål}$ i stålet**?

$$q := 1.0\ \frac{kN}{m} \qquad L_{AB} := 1\ m \qquad L_{BC} := 2\ m \qquad L_T := L_{AB} + L_{BC} = 3\ m$$

$$\Sigma M_C = 0$$

$$R_B := R_B \cdot L_{BC} - q \cdot (L_T) \cdot \left(\frac{L_T}{2}\right) = 0 \xrightarrow{explicit, ALL} R_B \cdot 2\ m - 1.0\ \frac{kN}{m} \cdot 3 \cdot m \cdot \frac{3 \cdot m}{2} = 0 \xrightarrow{solve, R_B} \frac{2.25 \cdot kN \cdot m^2}{m^2} = 2.25\ kN$$

$$\Sigma F_y = 0$$

$$R_C := -R_B + q \cdot (L_T) \rightarrow \frac{3.0 \cdot kN \cdot m}{m} - 2.25 \cdot kN = 0.75\ kN$$

Momment kurve A-B

$$M_{AB}(x) := -q \cdot x \cdot \frac{x}{2} \xrightarrow{explicit, ALL} \left(-\left(1.0\ \frac{kN}{m}\right)\right) \cdot x \cdot \frac{x}{2}$$

$$M_{maksAB} := M_{AB}(L_{AB}) \xrightarrow{explicit, ALL} \left(-\left(1.0\ \frac{kN}{m}\right)\right) \cdot 1\ m \cdot \frac{1\ m}{2} = -0.5\ kN \cdot m$$

Momment kurve B-C

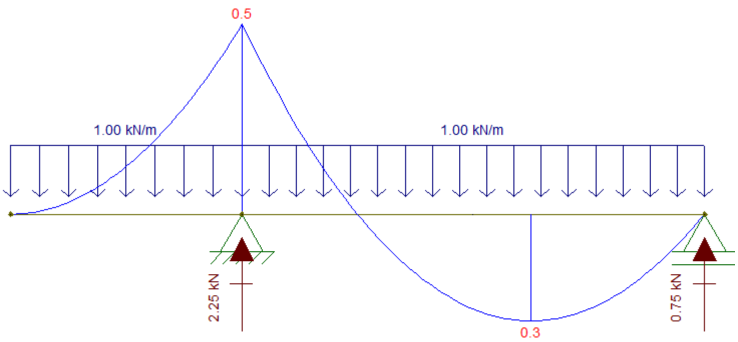
$$M_{BC}(x) := R_C \cdot (L_{BC} - x) - q \cdot \frac{(L_{BC} - x)^2}{2}$$

$$V_{BC}(x) := \frac{d}{dx} M_{BC}(x) \rightarrow -0.75 \cdot kN + \frac{0.5 \cdot kN \cdot (4 \cdot m - 2 \cdot x)}{m}$$

$$x_{maks} := 1.25\ m$$

$$V_{BC}(1.25\ m) \xrightarrow{explicit, ALL} -0.75 \cdot kN + \frac{0.5 \cdot kN \cdot (4 \cdot m - 2 \cdot 1.25\ m)}{m} = 0\ N$$

$$M_{maksBC} := M_{BC}(x_{maks}) \xrightarrow{explicit, ALL} 0.75 \cdot kN \cdot (2\ m - 1.25\ m) - 1.0\ \frac{kN}{m} \cdot \frac{(2\ m - 1.25\ m)^2}{2} = 0.281\ kN \cdot m$$



$$M_{maks} := \max\left(\left|M_{maksAB}\right|, \left|M_{maksBC}\right|\right) \rightarrow \max\left(0.5 \cdot \left|m \cdot kN\right|, 0.28125 \cdot \left|m \cdot kN\right|\right) = 0.5\ kN \cdot m$$

Spenning på overkant av stål

$$z_{stål\_topp} := h_{alu} + h_{stål} - z \xrightarrow{explicit, ALL} 50 \cdot mm + 20\ mm - 44 \cdot mm = 26.00\ mm$$

$$\sigma_{stål\_topp} := n \frac{M_{maks}}{I_t} \cdot z_{stål\_topp} \xrightarrow{explicit, ALL} 3 \frac{0.5 \cdot m \cdot kN}{2291300 \cdot mm^4} \cdot 26 \cdot mm = 17\ MPa$$

Spenning i stål på sted hvor stål koble med aluminium

$$\sigma_{stål\_alu} := n \frac{M_{maks}}{I_t} \cdot z_{topp\_alu} \xrightarrow{explicit, ALL} 3 \frac{0.5 \cdot m \cdot kN}{2291300 \cdot mm^4} \cdot 6 \cdot mm = 3.9\ MPa$$

$$\sigma_{maks} := \max\left(\left|\sigma_{stål\_alu}\right|, \left|\sigma_{stål\_topp}\right|\right) \rightarrow \max\left(3.9 \cdot \left|MPa\right|, 17 \cdot \left|MPa\right|\right) = 17\ MPa$$

c) Hva er **størst spenning σ_{alu} i aluminiumen**?

$$z_{topp\_alu} := h_{alu} - z \xrightarrow{explicit, ALL} 50 \cdot mm - 44 \cdot mm = 6\ mm$$

Spenning på overkant av aluminium

$$\sigma_{topp\_alu} := \frac{M_{maks}}{I_t} \cdot z_{topp\_alu} \xrightarrow{explicit, ALL} \frac{0.5 \cdot m \cdot kN}{2291300 \cdot mm^4} \cdot 6 \cdot mm = 1.3\ MPa$$

Spenning på underkant av aluminium

$$z_{bunn\_alu} := z = 44\ mm$$

$$\sigma_{bunn\_alu} := \frac{M_{maks}}{I_t} \cdot z_{bunn\_alu} \xrightarrow{explicit, ALL} \frac{0.5 \cdot m \cdot kN}{2291300 \cdot mm^4} \cdot 44 \cdot mm = 9.6\ MPa$$

$$\sigma_{maks} := \max\left(\left|\sigma_{topp\_alu}\right|, \left|\sigma_{bunn\_alu}\right|\right) \rightarrow \max\left(1.3 \cdot \left|MPa\right|, 9.6 \cdot \left|MPa\right|\right) = 9.6\ MPa$$

d) Hva er **maksimum skjærspenning** τ_{maks} mellom stål og aluminiumen?

Maksimum skjærkraft

Momment kurve A-B

$$M_{AB}(x) := -q \cdot x \cdot \frac{x}{2} \xrightarrow{explicit, ALL} \left(-\left(1.0 \frac{kN}{m}\right)\right) \cdot x \cdot \frac{x}{2}$$

Skjærkraft kurve A-B

$$V_{AB}(x) := \frac{d}{dx} M_{AB}(x) \rightarrow -\frac{1.0 \cdot x \cdot kN}{m}$$

Maks Skjærkraft A-B

$$V_{maksAB} := V_{AB}(L_{AB}) \xrightarrow{explicit, ALL} -\frac{1.0 \cdot 1\ m \cdot kN}{m} = -1.0\ kN$$

Momment kurve B-C

$$M_{BC}(x) := R_C \cdot (L_{BC} - x) - q \cdot \frac{(L_{BC} - x)^2}{2}$$

Skjærkraft kurve B-C

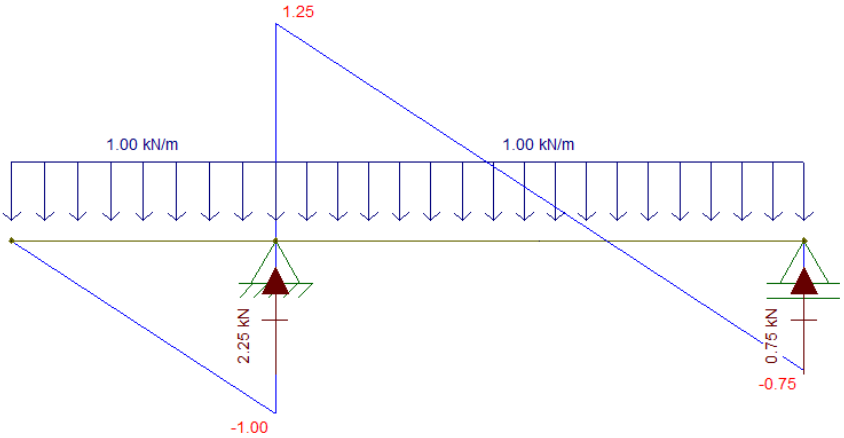
$$V_{BC}(x) := \frac{d}{dx} M_{BC}(x) \rightarrow -0.75 \cdot kN + \frac{0.5 \cdot kN \cdot (4 \cdot m - 2 \cdot x)}{m}$$

$$V_{BC}(0\ m) \xrightarrow{explicit, ALL} -0.75 \cdot kN + \frac{0.5 \cdot kN \cdot (4 \cdot m - 2 \cdot 0\ m)}{m} = 1.25\ kN$$

$$V_{BC}(L_{BC}) \xrightarrow{explicit, ALL} -0.75 \cdot kN + \frac{0.5 \cdot kN \cdot (4 \cdot m - 2 \cdot 2\ m)}{m} = -0.75\ kN$$

$$V_{maksBC} := \max\left(\left|V_{BC}(0\ m)\right|, \left|V_{BC}(L_{BC})\right|\right) \rightarrow \max\left(\left|-0.75 \cdot kN + 2.0 \cdot kN\right|, 0.75 \cdot |kN|\right) = 1.25\ kN$$

$$V_{maks} := \max\left(\left|V_{maksAB}\right|, \left|V_{maksBC}\right|\right) \rightarrow \max\left(|kN|, 1.25 \cdot |kN|\right) = 1.25\ kN$$



Skjærspenning mellom tverrsnitt (stål)

$$S_{stål} := b_{stål\_alu} \cdot h_{stål} \cdot (y_{stål}) \xrightarrow{explicit, ALL} 150 \cdot mm \cdot 20\ mm \cdot 15.9 \cdot mm = (4.77 \cdot 10^4)\ mm^3$$

$$\tau_{stål} := \frac{V_{maks}}{I_t \cdot b_{alu}} S_{stål} \xrightarrow{explicit, ALL} \frac{1.25 \cdot kN}{2291300 \cdot mm^4 \cdot 50 \cdot mm} (47700 \cdot mm^3) = 0.52\ MPa$$

Skjærspenning mellom tverrsnitt (aluminium)

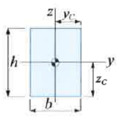
$$S_{alu} := b_{alu} \cdot h_{alu} \cdot (y_{alu}) \xrightarrow{explicit, ALL} 50 \cdot mm \cdot 50 \cdot mm \cdot 19.1 \cdot mm = (4.775 \cdot 10^4)\ mm^3$$

$$\tau_{alu} := \frac{V_{maks}}{I_t \cdot b_{alu}} S_{alu} \xrightarrow{explicit, ALL} \frac{1.25 \cdot kN}{2291300 \cdot mm^4 \cdot 50 \cdot mm} (47750.000000000007 \cdot mm^3) = 0.521\ MPa$$

Vedlegg 1: Arealer og Integrasjons tabell, Bjelkeformler

AREALER og VOLUMER

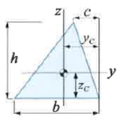
Symbolet ⊕ angir areal-/volum-senter.



Rektangel

$A = bh \qquad y_C = b/2 \qquad z_C = h/2$

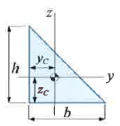
$I_y = \dfrac{bh^3}{12} \qquad I_z = \dfrac{hb^3}{12}$



Trekant

$A = bh/2 \qquad y_C = (b+c)/3 \qquad z_C = h/3$

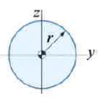
$I_y = \dfrac{bh^3}{36} \qquad I_z = \dfrac{bh}{36}(b^2 - bc + c^2)$



Rettvinklet trekant

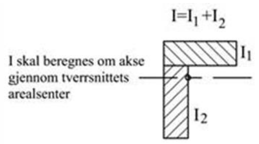
$A = bh/2 \qquad y_C = b/3 \qquad z_C = h/3$

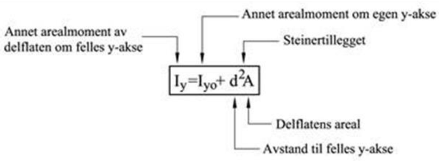
$I_y = \dfrac{bh^3}{36} \qquad I_z = \dfrac{hb^3}{36}$



Sirkel

$A = \pi r^2 \qquad I_y = I_z = \pi r^4/4$





Common Functions	Function	Integral
Constant	$\int a\,dx$	$ax + C$
Variable	$\int x\,dx$	$x^2/2 + C$
Square	$\int x^2\,dx$	$x^3/3 + C$
Reciprocal	$\int (1/x)\,dx$	$\ln\|x\| + C$
Exponential	$\int e^x\,dx$	$e^x + C$
	$\int a^x\,dx$	$a^x/\ln(a) + C$
	$\int \ln(x)\,dx$	$x\ln(x) - x + C$
Trigonometry (x in radians)	$\int \cos(x)\,dx$	$\sin(x) + C$
	$\int \sin(x)\,dx$	$-\cos(x) + C$
	$\int \sec^2(x)\,dx$	$\tan(x) + C$

Formler

$\sigma_x = \dfrac{M}{I_y} z$

$\tau_{maks} = \dfrac{V_y}{I_y b} S_y$

$z \Rightarrow$ avstanden fra nøytralaksen

$I_y \Rightarrow$ Arealmoment om nøytralaksen

$S_y \Rightarrow$ Statisk moment om nøytralaksen $\qquad S_y = \int_{A'} y\,dA = \sum z_i A_i$

$b \Rightarrow$ Bredde på profilet

BJELKEFORMLER

Her gjengir vi formler for enkle prismatiske bjelker basert på teknisk bjelketeori, dvs. EULER-BERNOULLI teori. Hovedvekten er lagt på forskyvninger og helninger (vinkler), men i noen tilfeller er også momenter og skjærkrefter gitt.

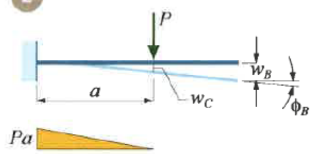
De aller fleste av formlene som er gjengitt kan utledes fra en av de to differensialligningene som ble utviklet i kapittel 5, dvs

$$\frac{d^4 w}{dx^4} = \frac{p(x)}{EI} \qquad \text{eller} \qquad \frac{d^2 w}{dx^2} = -\frac{M}{EI}$$

1

$$w_B = \frac{PL^3}{3EI} \qquad \phi_B = \frac{PL^2}{2EI}$$

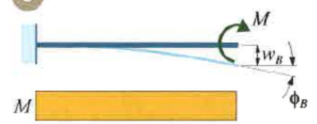
2



$$w_C = \frac{Pa^3}{3EI} \qquad w_B = \frac{Pa^2}{6EI}(3L-a)$$

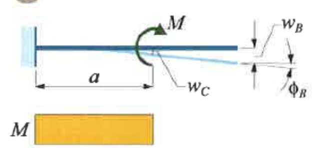
$$\phi_B = \frac{Pa^2}{2EI}$$

3



$$w_B = \frac{ML^2}{2EI} \qquad \phi_B = \frac{ML}{EI}$$

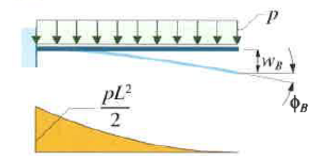
4



$$w_C = \frac{Ma^2}{2EI} \qquad w_B = \frac{Ma}{2EI}(2L-a)$$

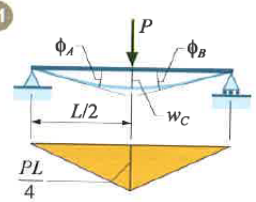
$$\phi_B = \frac{Ma}{EI}$$

5



$$w_B = \frac{pL^4}{8EI} \qquad \phi_B = \frac{pL^3}{6EI}$$

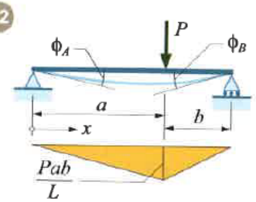
11



$$\phi_A = \phi_B = \frac{PL^2}{16EI}$$

$$w_C = \frac{PL^3}{48EI}$$

12

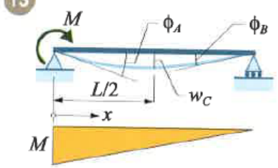


$$\phi_A = \frac{Pab(L+b)}{6L \cdot EI}$$

$$\phi_B = \frac{Pab(L+a)}{6L \cdot EI}$$

$$w(x) = \frac{Pbx}{6L \cdot EI}(L^2-b^2-x^2)\,, \quad x \le a$$

13

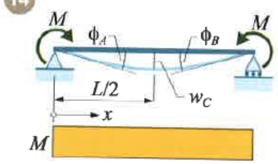


$$\phi_A = \frac{ML}{3EI} \qquad \phi_B = \frac{\phi_A}{2} = \frac{ML}{6EI}$$

$$w(x) = \frac{M(L-x)}{6L \cdot EI}(2Lx-x^2)$$

$$w_C = \frac{ML^2}{16EI}$$

14

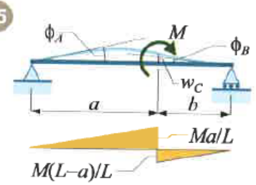


$$\phi_A = \phi_B = \frac{ML}{2EI}$$

$$w(x) = \frac{Mx}{2EI}(L-x)$$

$$w_C = \frac{ML^2}{8EI}$$

15

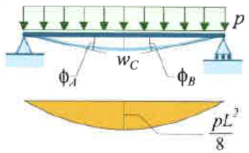


$$\phi_A = \frac{M}{6L \cdot EI}(6aL-3a^2-2L^2)$$

$$\phi_B = \frac{M}{6L \cdot EI}(3a^2-L^2)$$

$$w_C = \frac{Mab}{3L \cdot EI}(2a-L)$$

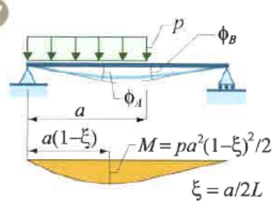
16



$$\phi_A = \phi_B = \frac{pL^3}{24EI}$$

$$w_C = \frac{5pL^4}{384EI}$$

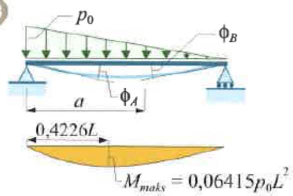
17



$$\phi_A = \frac{pa^2}{24L \cdot EI}(2L-a)^2$$

$$\phi_B = \frac{pa^2}{24L \cdot EI}(2L^2-a^2)$$

18

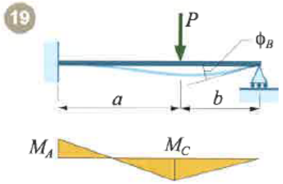


$$\phi_A = \frac{p_0L^3}{45EI}$$

$$\phi_B = \frac{7p_0L^3}{360EI}$$

$$w_{maks} = 0{,}00652\frac{p_0L^4}{EI} \quad \text{for } x = 0{,}4807L$$

19

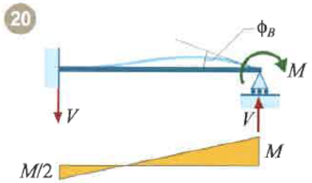


$$\phi_B = \frac{Pa^2b}{4L \cdot EI}$$

$$M_A = \frac{Pab}{2L}\left(1+\frac{b}{L}\right)$$

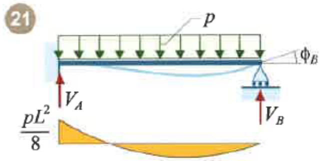
$$M_C = \frac{Pab}{2L}\left[2-\frac{b}{L}-\left(\frac{b}{L}\right)^2\right]$$

20



$$\phi_B = \frac{ML}{4EI} \qquad V = \frac{3M}{2L}$$

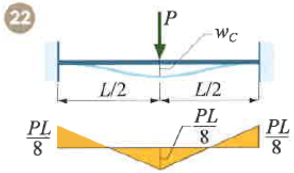
21



$$\phi_B = \frac{pL^3}{48EI}$$

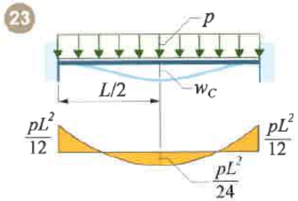
$$V_A = \frac{5pL}{8} \qquad V_B = \frac{3pL}{8}$$

22



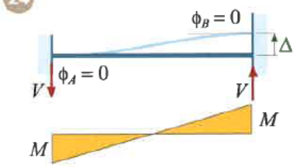
$$w_C = \frac{PL^3}{192EI}$$

23

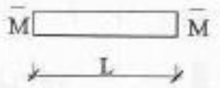
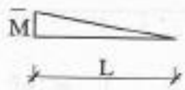
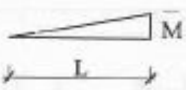
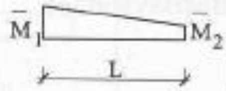
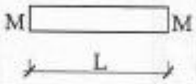
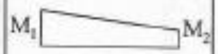
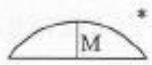
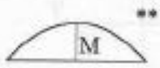
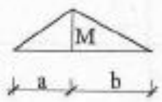


$$w_C = \frac{pL^4}{384EI}$$

24



$$M = \frac{6EI}{L^2}\Delta \qquad V = \frac{12EI}{L^3}\Delta$$

				
	$M\bar{M}L$	$\frac{1}{2}M\bar{M}L$	$\frac{1}{2}M\bar{M}L$	$\frac{1}{2}M(\bar{M}_1+\bar{M}_2)L$
	$\frac{1}{2}(M_1+M_2)\bar{M}L$	$\frac{1}{6}(2M_1+M_2)\bar{M}L$	$\frac{1}{6}(M_1+2M_2)\bar{M}L$	$\frac{1}{6}[M_1(2\bar{M}_1+\bar{M}_2)$ $+M_2(\bar{M}_1+2\bar{M}_2)]L$
	$\frac{1}{2}M\bar{M}L$	$\frac{1}{3}M\bar{M}L$	$\frac{1}{6}M\bar{M}L$	$\frac{1}{6}M(2\bar{M}_1+\bar{M}_2)L$
	$\frac{1}{3}M\bar{M}L$	$\frac{1}{4}M\bar{M}L$	$\frac{1}{12}M\bar{M}L$	$\frac{1}{12}M(3\bar{M}_1+\bar{M}_2)L$
	$\frac{2}{3}M\bar{M}L$	$\frac{5}{12}M\bar{M}L$	$\frac{1}{4}M\bar{M}L$	$\frac{1}{12}M(5\bar{M}_1$ $+3\bar{M}_2)L$
	$\frac{2}{3}M\bar{M}L$	$\frac{1}{3}M\bar{M}L$	$\frac{1}{3}M\bar{M}L$	$\frac{1}{3}M(\bar{M}_1+\bar{M}_2)L$
	$\frac{2}{\pi}M\bar{M}L$	$\frac{2\pi-4}{\pi^2}M\bar{M}L$	$\frac{4}{\pi^2}M\bar{M}L$	$\frac{2}{\pi^2}M[(\pi-2)\bar{M}_1$ $+2\bar{M}_2]L$
	$\frac{2}{\pi}M\bar{M}L$	$\frac{1}{\pi}M\bar{M}L$	$\frac{1}{\pi}M\bar{M}L$	$\frac{1}{\pi}M(\bar{M}_1+\bar{M}_2)L$
	$\frac{1}{2}M\bar{M}L$	$\frac{1}{6}(1+\frac{b}{L})M\bar{M}L$	$\frac{1}{6}(1+\frac{a}{L})M\bar{M}L$	$\frac{1}{6}M[\bar{M}_1(1+\frac{b}{L})$ $+\bar{M}_2(1+\frac{a}{L})]L$

* Momentforløpet er gitt ved et annengrads polynom (parabel)
** Momentforløpet er gitt ved en sinusfunksjon

Tabellen gir verdien av integralet: $\int_0^L M(x)\bar{M}(x)\,dx$